Legal Protection in Retail Financial Markets

Bruce Ian Carlin
University of California, Los Angeles

Simon Gervais
Duke University

We model a retail financial institution that outsources its advice services to an intermediary, making the two parties jointly responsible for consumers’ experience with the products. In this context, courts that enforce state-contingent legal rules are necessary in order to avoid market breakdowns. To maximize social welfare, the government implements a system of penalties that depends on product characteristics and on the firm’s relative ability to control quality. This legal system emphasizes reliable advice over transaction pace. Furthermore, the implicit team structure of the firm and its intermediary prevents self-regulation from achieving the same social efficiency. (JEL G18, G28, D11, D18, L51, K20)

Retail financial markets are unique in that the majority of consumers who participate have an incomplete understanding of the products that are available and are generally uninformed about prices in the industry (e.g., NASD Literacy Survey 2003). In fact, in the language of the Securities Act of 1933, public investors are described as those who are “unable to fend for themselves.” Participation at the household level,

We would like to thank Tony Bernardo, Doug Diamond, Denis Gromb, Raghu Rajan, Adriano Rampini, Jacob Sagi, and Andrei Shleifer for helpful discussions at the inception of this project. Also providing many useful comments and suggestions were Moez Bennouri, Utpal Bhattacharya, Philip Bond, Brendan Daley, Howell Jackson, Jakub Jurek, Rachel Kranton, Rich Mathews, Sébastien Michenaud, Eric Zitzewitz, two anonymous referees, the Executive Editor Paolo Fulghieri, as well as seminar participants at American University, Duke University (law school and economics department), Rice University, Texas A&M University, UCLA, the University of Miami, the University of Texas at Austin, the University of Virginia, the U.S. Securities and Exchange Commission, the UBC Summer Finance Conference, the 2009 Conference of the Financial Intermediation Research Society, the 2009 Mitsui Finance Symposium held at the University of Michigan, the 2009 Conference on Recent Developments in Consumer Credit and Payments held at the Federal Reserve Bank of Philadelphia, the 2009 Conference on the Role of Government Regulation in Corporate Finance held at Vanderbilt University, the NBER Working Group on Household Finance, the NBER Working Group on Law and Economics, and the 2010 Annual Meeting of the American Finance Association. Send correspondence to Bruce Ian Carlin, Anderson School of Management, University of California, Los Angeles, 110 Westwood Plaza Suite C413, Los Angeles, CA 90095; telephone: (310) 825-7246. E-mail: bruce.carlin@anderson.ucla.edu; or to Simon Gervais, Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708-0120; telephone: (919) 660-7683. E-mail: sgervais@duke.edu.
therefore, not only involves having access to good-quality opportunities, but also entails being directed toward the best alternatives.

There is no clear evidence that advice increases welfare, however (e.g., Bergstresser, Chalmers, and Tufano 2009; Bhattacharya et al. 2012). As argued by Bolton, Freixas, and Shapiro (2007), this may be due to conflicts of interest and can be a significant cause of decreased faith in the market. Given the large size of retail markets, protecting consumers who are “unable to fend for themselves” is not only an important duty of the law, but also a key driver of participation in the market, economic growth, and social welfare (e.g., Calvet, Campbell, and Sodini 2006; Campbell 2006). The importance of regulation in such markets has been highlighted and described by Tufano (2009) and Campbell et al. (2011), but there is still a paucity of theoretical work on this subject.\footnote{We review this work in Section 1.}

In this article, we provide a theoretical analysis of consumer protection laws, taking into consideration that financial institutions frequently outsource their advice services to brokers. Indeed, the majority of financial products are sold through intermediaries. For example, only 40% of mutual funds are purchased directly from financial institutions (Investment Company Institute 2003). As discussed by Jackson (2008), this trilateral arrangement between the consumer, the intermediary, and the producer of the financial service complicates the legal structure that must be implemented to preserve efficiency. For one thing, when a household investor is wronged in the market for financial products, two parties are potentially culpable: the producer (e.g., a bank or a mutual fund family) and the intermediary (e.g., an advisor or broker). Thus, any law that is implemented must take into account the potential actions of the producer regarding product quality (e.g., transparency, cost effectiveness), the actions of the advisors in placing clients into those instruments (e.g., responsible vs. irresponsible advice), and the contractual arrangements between these parties.

In our model, a single firm produces two financial products and distributes them to the public through a broker who provides advice services. Half of the customers are better suited for one of the products, and the other half are better suited for the other. Each product can yield a good outcome or a bad outcome, but bad outcomes result in severe losses for customers who are not properly matched to their optimal product. In this context, the firm can choose to improve the quality of its products and thereby increase the probability of good outcomes. The broker’s job is to attract clients and to match them with products, based on noisy signals that he gathers about their optimal product. The broker controls the precision of these signals, knowing that more precise information

---

1 We review this work in Section 1.
improves the likelihood of a match but reduces the number of customers that he can advise. How the firm and the broker fulfill their responsibilities is unobservable and non-verifiable, and so both parties face the coordination problems that arise in settings with bilateral hidden action.\(^2\)

Our analysis starts with a characterization of the equilibrium in the absence of regulation or courts. In this case, neither the firm nor the broker can commit to provide quality or advice. Any level of quality or advice anticipated by customers is met with an opportunity to shirk and pocket larger profits by the firm and agent. Since consumers are rational, they anticipate the absence of quality and advice and, as a result, the market breaks down and no consumer surplus is ever realized.

We then introduce courts, which allow for the verification and enforcement of contracts that condition on states of the world in which consumers suffer large losses with unsuitable (i.e., mismatched) products. At first, the use of state-dependent monetary transfers is limited to the government. Specifically, the government maximizes total welfare by setting a law that holds the firm and the broker responsible when a consumer is wronged and suffers a loss. Government regulation makes optimal use of advice services in the market and splits the blame between the firm and the broker, both of whom have a fiduciary responsibility to the consumer.

When the firm’s and agent’s penalties can be transferred to consumers without friction, the government is able to implement first-best actions and to recover the full economic surplus. When transfers enacted by the court system come with (proportional) deadweight losses, it is impossible to attain first-best actions and welfare through government regulation. In this case, it becomes optimal for the government to implement a conservative regime in which fewer products are sold with better guidance than in first-best.

The size of the penalties and the balance of blame between the firm and the broker rest on the product characteristics and on the joint ability of the firm and agent to improve the customers’ experience. For example, when products are more generic (specialized), the government imposes smaller (larger) penalties, but shifts the blame toward the broker (firm). Thus, laws should favor punishing financial institutions more when products are complex and more investor-specific (e.g., high-yield corporate bond funds, adjustable-rate mortgages, annuities), whereas advisors should be held more accountable when products have widespread appeal (e.g., S&P 500 index funds, fixed-rate mortgages, life insurance). Similarly, when products can generate larger positive outcomes, the government finds it optimal to decrease penalties but attribute more of the blame to the broker. Finally, when the firm’s ability to improve quality

\(^2\) Similar settings of bilateral moral hazard are identified and discussed by Levmore (1993).
increases, the government decreases the total penalty transfers to wronged consumers and puts more blame on the broker for mismatches.

The last portion of the article introduces the possibility for the firm to write state-contingent contracts with consumers. Since the role of courts is to enforce such contracts, no matter who wrote them, it is theoretically possible for the firm to imitate the choices of the government and implement the maximum welfare equilibrium. Our analysis shows that the firm’s choices are different from those of the government, and that this is the direct result of the implicit team structure that prevails between it and the broker. Specifically, because the firm also controls the compensation contract of the agent, it finds it optimal to reduce the broker’s profits by lowering both his penalties and his sales commissions. The resulting equilibrium is one with more sales, less advice, and lower prices than in first-best. That is, to maximize its own profits, the firm prefers a large transaction volume with more modest transfers to wronged consumers. This is socially suboptimal: in essence, the firm prefers a larger piece of a smaller pie.

As suggested by Campbell et al. (2011), the law plays a crucial role in retail financial markets because other commitment devices and market discipline devices are often inadequate in this setting. For example, because financial products are inherently risky and it is difficult to measure their ex ante suitability based on ex post outcomes, it is usually unfeasible to offer individual consumer protection on such products, as this would engender unsurmountable adverse-selection problems. Indeed, in our analysis, such warranties are not sufficient for full efficiency, as the firm uses them to reduce the advice channel and redirect economic surplus in its direction, not to maximize welfare. Thus, whereas Spence (1977), Grossman (1981), and Mann and Wissink (1990) suggest that warranties and refunds can restore efficient outcomes, commitment to quality via such mechanisms is unreliable in retail financial markets. As such, we expect the legal system to play a more significant role in these markets, as in Palfrey and Romer’s (1983) analysis of disputes over product performance between buyers and sellers.

Reputation is also insufficient to induce full commitment to quality or advice, as proposed by Klein and Leffler (1981), Shapiro (1982, 1983), and Allen (1984). The reason is that products and prices in these markets

---

3 This result is reminiscent of that by DeMarzo, Fishman, and Hagerty (2005) in a different context. Specifically, they show that the incentives of a self-regulatory organization (SRO) to investigate and punish the fraudulent activities of its members will differ from those of a government seeking to maximize customer expected utility. Their result derives from the fact that lax regulation allows the SRO to reduce the competition across its members at the expense of consumers. In contrast, ours is related to the fact that one member (the firm) has more control over the split of economic surplus with the other member (the broker).
are inherently difficult for consumers to decipher. As a result, consumers often settle on a suboptimal product, as documented by Capon, Fitzsimmons, and Prince (1996), Agnew and Szykman (2005), Agarwal et al. (2009), and Choi, Laibson, and Madrian (2010), among many others. Moreover, as shown by Ausubel (1991), Jain and Wu (2000), Jones and Smythe (2003), and Choi, Laibson, and Madrian (2004) in different contexts, these consumers are frequently unable to discriminate among brokers and providers of services, due to various constraints on their discovery processes (e.g., ability or cost to learn). Finally, the low frequency with which the average consumer interacts with a financial product provider seriously limits the efficiency of reputation-building as a disciplining device, especially when the transactions and experiences of other market participants are not publicly observable.4

Our article contributes to a growing theoretical literature on household finance and agency law. We highlight this contribution in the next section. Following this, in Section 2, we set up the model and show the necessity of regulation for any surplus to be realized. Section 3 derives the equilibrium of the model. In turn, we analyze optimal consumer behavior, the strategic choices of the firm and broker, and the government’s optimal choice of regulation. Section 4 shows how the team structure of the firm and the broker renders self-regulation suboptimal, even if the firm can make use of state-contingent contracts that courts subsequently enforce. Section 5 offers some concluding remarks. The Appendix contains all the proofs.

1. Related Literature

Our article contributes to the theoretical literature on household finance, in which rational financial institutions interact with heterogeneous consumers who rationally participate in the market, but must make decisions based on a constrained learning process.5 Whereas consumers are assumed to have incomplete knowledge about prices in Carlin (2009) and about the quality of products in Carlin and Manso (2011), we assume instead that consumers have limited information about the appropriateness of a specific financial product for their own situation. In this context, consumers benefit not only from a higher commitment to quality by the firm, but also from the advice of the agent hired by the firm to match products and customers. As in

---

4 In fact, it could be argued that a legal system is necessary for a reputation to form in these markets. That is, given poor access to information, the presence of lawsuits acts as a device for reputation to form and get disseminated in the population. We leave this additional role for the legal system to future research.

5 This literature has evolved from the initial insight of Stigler (1961) about price dispersion and the subsequent consumer models of Shilony (1977), Varian (1980), and Burdett and Judd (1983).
Kronman’s (1978) discussion of voluntary disclosures and in Shavell’s (1994) model of the same problem, the presence of legal obligations changes the agent’s incentives to gather and communicate information that is socially useful. Our analysis adds the aforementioned tensions between the firm and the agent to this problem, characterizes their contractual relationship, and derives the regulation that maximizes economic welfare.

By including a layer of agency between the producer of the product and the eventual consumer, our work is closely related to recent papers by Inderst and Ottaviani (2009a, 2011, 2012). In particular, the fact that agents may oversell or over-recommend a product when their compensation includes sales commissions and kickbacks is a force that is present in our and all of their papers. The main differences across the papers relate to the set of instruments that are available to the firm and to regulatory bodies in order to limit the surplus that is lost to agency frictions. For example, Inderst and Ottaviani (2009a) emphasize the role of penalties imposed on the producer in mitigating the mis-selling incentives that are ultimately provided to the agent. Our model complements this analysis by incorporating the possibility that the agent and the firm can free-ride on each other, so that penalties imposed on only one party make the other party more difficult and costly to restrain. As such, it is the optimal mix of penalties that becomes our article’s main concern. Also, whereas Inderst and Ottaviani (2011, 2012) allow for kickbacks and disclosure rules when producers compete with each other or when agents are potentially naïve, we restrict our analysis to one producer, one agent, and rational (but uninformed) consumers, as in Inderst and Ottaviani (2009a).

In our model, consumers are not as adept as brokers in assessing the suitability of products for their own needs. This makes financial products similar to credence goods which, as originally described by Darby and Karni (1973) and as surveyed by Dulleck and Kerschbamer (2006), include services and products whose endogenous provision may be affected by the seller’s incentives (e.g., an auto repair that is unnecessary). The market for financial products that we study differs in key respects from that for credence goods. First, the quality of the financial product cannot be verified by any kind of hard information. Second, whereas the quality input of the producer becomes an intrinsic property of a credence good purchased by the consumer, the quality of a financial product is typically an ongoing concern. That is, the producer’s choice of quality affects the financial product after the transaction (e.g., an investment in a mutual fund), limiting the set of instruments that can be used to realign incentives. Third, the risky nature of financial products implies that even good products sold to properly advised consumers may end up not performing as well as anticipated. These differences, as we show, are especially
important when sales are intermediated by agents who can advise customers.\textsuperscript{6}

Our article also adds to the literature on law and finance, which highlights the link between strong legal and financial institutions and economic growth. For example, Shleifer and Wolfenzon (2002) analyze the effects that legal protection has on the type and quality of investments that occur in the market. Similarly, Stulz (2009) shows how strong securities laws that mandate disclosures can significantly impact firms’ access to capital and their value, as suggested by La Porta, Lopez-de-Silanes, and Shleifer (2006). This work underscores several empirical observations that there is a strong relationship between legal institutions and economic progress. Indeed, La Porta et al. (1997, 1998) document substantial cross-sectional variation in the legal protection that investors receive in different countries, and posit that there exists a positive correlation between government regulation and economic growth. Levine (1999), Glaeser, Johnson, and Shleifer (2001), and Beck, Demirgüç-Kunt, and Levine (2005) also argue for this positive relationship. In a similar vein, Levine (1998), Levine, Loayza, and Beck (2000), and Haselmann, Pistor, and Vig (2010) provide evidence that financial intermediation and the provision of credit are greatly affected by the legal system, while Nunn (2007) shows that the ability of a legal system to enforce contracts is a significant driver of economic activity.\textsuperscript{7} Consistent with these empirical observations, the analysis in this article demonstrates that consumer protection law is necessary for both the preservation and the prosperity of retail financial markets. In this sense, our work complements that of Acemoglu, Antrás, and Helpman (2007), who show that strong contract enforcement facilitates the adoption of more advanced technology.

By theoretically investigating the optimal division of blame between the firm and the broker, our article may also be viewed as an economic analysis of agency law (e.g., Rasmusen 2004). Indeed, following Ross (1973), Jensen and Meckling (1976), and Holmström (1979), economists have focused their study of agency theory on the search for contractual arrangements that realign the incentives of agents with those of the principal, thereby maximizing the production potential and value of the firm. In contrast, and as laid out by Sykes (1984, 1988), the primary objective of legal scholars studying agency law is to determine who is to blame (principal or agent, or both) when an outsider is wronged. As Rasmusen (2004, p. 370) writes, “for the economist, the agency problem is how to

\textsuperscript{6} A similarity between credence goods and financial products is that the moral hazard problems associated with them are unlikely to be eliminated by reputation concerns alone. For example, Schneider (forthcoming) empirically documents that reputation does not seem to restore much efficiency in the market for auto repair.

\textsuperscript{7} For a comprehensive overview of the literature on law and economic growth, see La Porta, Lopez-de-Silanes, and Shleifer (2008).
give the agent incentives for the right action; for the lawyer, it is how to ‘mop up’ the damage once the agent has taken the wrong action.”

Finally, our article is closest in spirit to recent discussion papers by Barr, Mullainathan, and Shafir (2008) and Lipner and Catalano (2009) about the regulation of home mortgage credit and negligent investment advice respectively. Like us, Barr, Mullainathan, and Shafir argue that the complexity of the decisions that consumers are asked to make about mortgages requires a legal system that properly internalizes the incentives, motives, and biases of market participants. Similarly, Lipner and Catalano advocate a system of legal responsibility that fills the gaps in existing securities statutes and holds brokers and advisors accountable for “negligent misrepresentation.” Our article complements these papers by providing an economic analysis that formalizes the main ideas, explicitly characterizes the economic forces, and extends their applicability to the entirety of retail financial markets.

2. A Model of Retail Finance

2.1 Model setup

Consider a risk-neutral financial institution (the firm) that markets two financial products, $X$ and $Y$, to a unit mass of consumers, who are indexed by $i$. The products might be used to finance the purchase of consumption goods (e.g., credit cards or mortgages) or used as investment vehicles (e.g., mutual funds). The two products are mutually exclusive in the sense that each consumer must choose one or the other (e.g., a fixed-rate mortgage vs. an adjustable-rate mortgage). Consumer $i$’s experience with either product $i \in \{X, Y\}$ can be good ($\tilde{r}_i = G$) or bad ($\tilde{r}_i = B$), and the ex ante probability of these outcomes is $\phi \in [\frac{1}{2}, 1)$ and $1 - \phi$, respectively. We can think of $\phi$ as measuring how generic the products are: when $\phi$ is large, most products generate good outcomes for most consumers.

The firm can choose one of two quality levels for its products: $q \in \{0, \tilde{q}\}$, with $\tilde{q} \in (0, 1)$. This choice is unobservable, but $q = \tilde{q}$ is known to cost the firm $k_F$ per product sold.8 When the firm chooses $q = \tilde{q}$, it increases the probability that its products yield a positive outcome from $\phi$ to $\phi + (1 - \phi)\tilde{q}$. For example, this quality input by the firm can be thought of as the skill of the people it hires to manage funds, the internal monitoring of its activities, or the care that goes into product development. The fact that the cost of quality increases with the number of sales is meant to capture the idea that the total costs of skill,

---

8 The assumption that quality can take only two possible values does not affect or lead to our results. However, it greatly enhances tractability and in particular allows us to derive all of our results in closed form.
monitoring, and care are bound to increase with the size of the firm’s operations.\(^9\)

Consumers experience a positive utility of \(h\) when the product they purchase generates a good outcome. However, their capacity to deal with bad outcomes is different across the two products. Specifically, a consumer \(i\) is *fit* for only one product \(\tilde{\tau}_i \in \{X, Y\}\), and we refer to this product as his type. We say that customer \(i\) is *unfit* for the other product. We assume that the utility of a customer with a product that performs badly is zero when he is fit for it, and \(-\ell < 0\) when he is unfit for it. For example, it may be that an inappropriately chosen product that generates a bad outcome forces a customer into a costly bankruptcy process.

Sales of the firm’s products to consumers are intermediated by a risk-neutral broker (the *agent*). The broker’s role is to attract customers and to match them to one of the firm’s two products. As such, there is a division of labor in which the firm is responsible for producing the good, while the broker is responsible for its distribution and for providing potential clients with financial advice. For example, a mutual fund provider relies on a broker to highlight the merits of its products and to guide customers toward the fund that is most appropriate for them.

Specifically, the agent chooses two quantities: the fraction \(n \in [0, 1]\) of consumers that he attracts, and the level of advice \(a \in [0, 1]\) that he provides to these consumers. We normalize the utility of consumers who fail to be attracted by the broker to zero. We let \(U_i(\tilde{\tau}_i, t)\) denote the utility of any consumer \(i \in [0, n]\) who is attracted by the broker and who buys product \(t \in \{X, Y\}\). These consumers are unaware of their own type, but do know that \(\Pr[\tilde{\tau}_i = X] = \Pr[\tilde{\tau}_i = Y] = \frac{1}{2}\). In particular, they know that their expected utility from owning product \(t\) without quality input from the firm and without any advice (which we introduce below) is

\[
E[U_i(\tilde{\tau}_i, t)] = \Pr[\tilde{\tau}_i = t][\Pr[\tilde{\tau}_i = G]h + \Pr[\tilde{\tau}_i = B] \cdot 0] \\
+ \Pr[\tilde{\tau}_i \neq t][\Pr[\tilde{\tau}_i = G]h + \Pr[\tilde{\tau}_i = B](-\ell)] \\
= \frac{1}{2} \phi h + \frac{1}{2} [\phi h - (1 - \phi)\ell] = \phi h - \frac{1}{2} (1 - \phi)\ell. \tag{1}
\]

We assume throughout the article that

\[
\frac{h}{\ell} < \frac{1 - \phi}{2\phi}, \tag{2}
\]

so that (1) is strictly smaller than zero, the utility that a customer experiences without buying either product.\(^{10}\) This ensures that, without

---

\(^9\) The assumption that these costs are proportional to sales preserves tractability and closed-form expressions. Also, although a fraction of these costs are incurred at the time the firm creates the products, some of the costs are incurred over time, even if they depend on quality decisions made earlier by the firm.

\(^{10}\) Actually, for reasons to be made clear later, we will assume a stricter inequality in (a1) below.
knowing their type, customers attracted to the firm are not willing to pay a positive price for either product unless a sufficient amount of quality and advice improves their expected performance.

In the spirit of Inderst (2008) and Inderst and Ottaviani (2009a), advertising and advising are substitutes in that effort exerted on one function reduces the effort that can be spent on the other. That is, we assume that the agent’s total effort capital is $k_A > 0$ and that this capital must be allocated to increasing $n$ (i.e., advertising) or increasing $a$ (i.e., advising).

Without loss of generality, we set $k_A = 1$. Thus, the agent must choose $n$ and $a$ such that $n + a \leq 1$, a constraint that always binds since the agent is always better off using all the capital at his disposal.

The advising function of the agent allows him to gather information about the type $\tilde{\tau}_i$ of every customer $i \in [0, n]$ that he attracts, and to recommend one of the firm’s two products, $X$ or $Y$. This information takes the form of a signal $\tilde{s}_i = \delta_i \tilde{\tau}_i + (1 - \delta_i) \tilde{\eta}_i \in \{X, Y\}$, where $\tilde{\eta}_i$ has the same distribution as $\tilde{\tau}_i$ but is independent from it, and

$$\tilde{\delta}_i = \begin{cases} 1, & \text{prob. } a \\ 0, & \text{prob. } 1 - a. \end{cases}$$

Since $\tilde{\eta}_i$ is uninformative about $\tilde{\tau}_i$, larger values of $a$ allow the agent to better match customers and products. Indeed, since $\Pr[\tilde{s}_i = t \mid \tilde{\tau}_i = t] = 1$, $\Pr[\tilde{s}_i = 1] + \frac{1}{2} \Pr[\tilde{s}_i = 0] = a + \frac{1-a}{2} = \frac{1+a}{2}$, the signal received by the agent is more likely to be the consumer’s type when $a$ is large. The agent updates his beliefs about $\tilde{\tau}_i$ based on the signal that he receives. Lemma 1 characterizes this updating process.

**Lemma 1**

*Suppose that the agent chooses a level $a \in [0, 1]$ of advice. After he observes $\tilde{s}_i = t \in \{X, Y\}$, he updates the probability that customer $i$ is fit for product $t$ to*

$$\Pr[\tilde{\tau}_i = t \mid \tilde{s}_i = t] = \frac{1 + a}{2}. \tag{3}$$

The broker cannot directly observe the firm’s choice of $q$ and, likewise, the firm cannot directly observe the broker’s choices of $a$ and $n$. The resulting model is one of bilateral hidden action in which both parties are rational and have consistent beliefs about each other’s equilibrium behavior. The agent’s advice guides consumers’ product choices, but it is impossible for the agent to tell consumers about how resources and products are managed internally within the firm. Similarly, the firm chooses

11 Also, it will be clear from the analysis that follows that the agent is always better off recommending product $\tilde{s}_i$ to consumer $i$.  

---

77
how much capital it expends to add quality to its investment products or services, but cannot oversee how the agent divides his effort capital between attracting and advising consumers.

The incentives of the firm and agent are guided by two forces: the compensation contracts that are written between the firm and the agent, and the legal environment imposed by the government. We assume that the firm compensates the agent via sales commissions. As such, the firm sets the amount \( w \geq 0 \) that the agent receives for each sale. We also assume that the agent’s wealth and reservation utility are both zero at the outset. This rules out any fixed-wage compensation, as the agent cannot make any fixed payment to the firm and any fixed payment to the agent is a deadweight loss to the firm.

The legal environment is set as follows. We assume that a court of law can perfectly verify whether or not a customer has experienced a loss as a result of a bad outcome for a product that he purchased. Such verification clearly indicates the presence of mis-selling (to use Inderst and Ottaviani’s 2009a terminology): the product was sold to a customer who was unfit for it and who suffered following a bad outcome. To protect consumers, the legal system set by the government imposes penalties of \( x_A \geq 0 \) and \( x_F \geq 0 \) on the agent and the firm, respectively. To capture the possibility that the legal process is costly, we assume that a fraction \( \lambda \in [0, \frac{1}{2}] \) of the penalties are lost to frictions; that is, a wronged consumer receives only \((1 - \lambda)(x_A + x_F)\) and the balance, \(\lambda(x_A + x_F)\), represents a deadweight loss. In setting the legal environment \( L = \{x_F, x_A\} \), the government’s objective is to maximize total welfare in the economy, net of any deadweight losses that are associated with penalties.

An important difference between the two forces that affect the incentives of the firm and agent is the knowledge that consumers have about them. Although the legal environment announced by the government is

---

12 The possibility of transfer payments that are contingent on the consumers’ experience with the products is analyzed in Section 4.

13 Since the firm and agent are risk-neutral, the assumption that this verification is perfect is without loss of generality, as the penalties could be appropriately scaled if verification were imperfect. Specifically, a verification mechanism that identifies an agent’s utility realization of \(-\ell\) with probability \( \alpha \in (0, 1] \) would lead to the same equilibrium if the penalties described below were set at \( x_A' = \frac{x_A}{\alpha} \) and \( x_F' = \frac{x_F}{\alpha} \).

14 As we will see later in the proof of Proposition 2, our assumptions that \( \phi \geq \frac{1}{2} \) (from earlier) and that \( \lambda \leq \frac{1}{2} \) ensure that the government maximizes social welfare with \( n > 0 \). Otherwise, the large and frequent frictional losses may push the government into closing the market altogether.

15 Note that we deliberately assume that penalties are not affected by the number of consumers who end up suing. By making each customer’s penalty independent from the penalties of others, we capture the idea that in reality transactions for financial products and the lawsuits that result from them do not all occur at the same time, as they do in the model. Instead, each consumer can appeal to the court system if and when they are wronged, regardless of what is likely to happen subsequently to others. In fact, in this light, our assumption is consistent with that of Inderst and Ottaviani (2009a), who consider only one consumer. Indeed, given that the firm and agent are risk-neutral, \( \alpha \) and \( \beta \) then jointly determine the probability that this one consumer sues, without affecting the analysis.
public information, the compensation contract between the firm and the agent is private to these two parties. As we shall see, this difference creates room for opportunistic behavior by the firm when it sets $w$.

Besides setting the agent’s compensation contract, the firm also sets the price $p$ at which products are sold to consumers. In setting this price, the firm anticipates the equilibrium beliefs that consumers form about the equilibrium actions of the firm and the agent. Ultimately, consumers relinquish all the surplus from the transaction to the firm and agent. Of course, this has no effect on total welfare, as the price at which products are sold to consumers is a transfer between two market participants. Still, the willingness of consumers to pay higher prices will clearly indicate a better-functioning market for retail financial products.

The timing of the game is outlined in Figure 1. At $t = 1$, the law is set. At $t = 2$, the broker is hired with a contract that pays $w$ per sale, and the price for the financial product is posted by the firm. At $t = 3$, the broker jointly chooses $n$ and $a$, while the firm chooses $q$. Finally, at $t = 4$, all outcomes and payoffs are realized, and consumers who suffer a loss sue for damages based on the law.

---

**Figure 1**
Sequence of events. At $t = 1$, the law is set. At $t = 2$, the broker is hired with a contract that pays $w$ per sale, and the price for the financial product is posted by the firm. At $t = 3$, the broker jointly chooses $n$ and $a$, while the firm chooses $q$. Finally, at $t = 4$, all outcomes and payoffs are realized, and consumers who suffer a loss sue for damages based on the law.

---

16 By symmetry, the two products must always have the same price in equilibrium.

17 Our one-period model does not facilitate a study of early cancellation, in which a seemingly underperforming product is returned to the seller, who then issues a pre-contracted refund. This problem is studied by Inderst and Ottaviani (2009b).
2.2 First-best

Before we proceed to study the equilibrium behavior of all the economy’s participants, let us characterize the first-best scenario in this economy. This scenario imposes welfare-maximizing choices of \( n \), \( a \), and \( q \) on the agent and firm, and will later serve as a benchmark for the ability of the government to align incentives through its choice of a legal system.

Consider a consumer’s decision after being attracted and advised with \( \tilde{s}_i = t \in \{X, Y\} \) by the broker. If he chooses product \( t \), his expected utility is given by

\[
E[U_i(\tilde{s}_i, t) | \tilde{s}_i = t] = \text{Pr}(\tilde{r}^G_i = G)h + \text{Pr}(\tilde{r}^B_i = B) \text{Pr}(\tilde{r}_i \neq t | \tilde{s}_i = t)(-\ell)
\]

\[
= [\phi + (1 - \phi)q]h - (1 - \phi)(1 - q) \frac{1 - a}{2} \ell. \tag{4}
\]

If instead customer \( i \) decides not to follow the broker’s advice and to buy product \( t' \neq t \), then his utility is

\[
E[U_i(\tilde{s}_i, t') | \tilde{s}_i = t] = \text{Pr}(\tilde{r}^G_i = G)h + \text{Pr}(\tilde{r}^B_i = B) \text{Pr}(\tilde{r}_i \neq t' | \tilde{s}_i = t)(-\ell)
\]

\[
= [\phi + (1 - \phi)q]h - (1 - \phi)(1 - q) \frac{1 + a}{2} \ell. \tag{5}
\]

A comparison of (4) and (5) clearly establishes that the consumer is better off following the broker’s advice. Thus, when choosing between two equally priced products, the consumer always picks product \( \tilde{s}_i \).

Figure 2 illustrates the expected sorting of consumers attracted by the agent in this economy. Half the consumers are advised to purchase each product, based on the signal \( \tilde{s}_i \) that the agent receives about their type.\(^{18}\)

When the product they purchased performs well \((\tilde{r}^G_i = G)\), these consumers experience a utility of \( h > 0 \). When it does not, then those who turn out to be fit (unfit) for the product experience a utility of zero (of \( -\ell < 0 \)).

---

\(^{18}\) Note that, because \( \tilde{r}_i \) and \( \tilde{s}_i \) share the same distribution, the unconditional distribution of \( \tilde{s}_i \) is also the same, and so \( \text{Pr}(\tilde{s}_i = X) = \text{Pr}(\tilde{s}_i = Y) = \frac{1}{2} \).
In this economy, total welfare is measured by the utility gains and losses of consumers, net of the costs that the firm’s quality input necessitates. Thus, since the consumers who are not attracted by the broker receive a utility payoff of zero, the social planner seeks to solve the following maximization problem:

$$
\text{max}_{n,a,q} \mathcal{W}_0 = \int_0^1 E[U_i(\tilde{x}_i, \tilde{s}_i) | \tilde{s}_i] di - nk_F 1_{[q=q]} \\
= n\left( [\phi + (1 - \phi)q]h - (1 - \phi)(1 - q) \frac{1 - a}{2} \ell - k_F 1_{[q=q]} \right)
$$

(6)

subject to \( n \in [0, 1], \quad a \in [0, 1], \quad n + a \leq 1, \quad q \in [0, \bar{q}] \).

In order to avoid corner solutions for \( n \) and \( a \), we assume that

\[
\frac{h}{\ell} \leq \frac{(1 - \phi)(1 - \bar{q})}{2[\phi + (1 - \phi)\bar{q}]}.
\]

(a1)

This assumption implies that, when \( a = 0 \), (6) is negative for any \( k_F > 0 \) and any \( q \in [0, \bar{q}] \). Thus, quality alone is not enough to improve social welfare; reliable advice is also necessary. It is easy to verify that \( \mathcal{W}_0 > 0 \) if only if

\[
a \geq 1 - \frac{2(h[\phi + (1 - \phi)q] - k_F 1_{[q=q]})}{\ell(1 - \phi)(1 - q)} \equiv \bar{a}_q.
\]

(7)

We further assume that \( \bar{a}_q < \bar{a}_0 \), which is equivalent to

\[
k_F < h\bar{q}.
\]

(a2)

This second assumption implies that welfare improvement requires more advice when the firm chooses not to improve the quality of its products.

It is easy to verify that \( \mathcal{W}_0 \) is increasing in \( n \) and in \( a \), and so it is always the case that \( n + a = 1 \). We can therefore rewrite the above maximization problem more succinctly as follows:

\[
\text{max}_{n \in [0, 1], q \in [0, \bar{q}]} \mathcal{W}_0 = n\left( [\phi + (1 - \phi)q]h - \frac{1}{2} (1 - \phi)(1 - q)n\ell - k_F 1_{[q=q]} \right).
\]

(8)

Proposition 1 characterizes the solution to this problem.

**Proposition 1 (First-Best)**

*In first-best, it is optimal to set*

\[
n = \frac{h[\phi + (1 - \phi)q] - k_F 1_{[q=q]}}{\ell(1 - \phi)(1 - q)}
\]

(9)

19 The indicator function \( 1_E \) takes a value of one when \( E \) is true and zero otherwise.
\[ a = 1 - n, \quad (10) \]

where \( q = \bar{q} \) if and only if

\[ k_F \leq h\left[ \phi(1 - \sqrt{1 - \bar{q}}) + (1 - \phi)\bar{q} \right] \equiv \bar{k}_0. \quad (11) \]

As mentioned above, every customer attracted by the broker ends up buying a product. Thus, from (9), we see that the optimal number of sales is increasing in the firm’s quality input \( \bar{q} \) (when the firm chooses \( q = \bar{q} \)), in how generic the products are (\( \phi \)), and in the potential utility gain that customers get from the products (\( h \)). It is decreasing in the potential losses that customers experience when they are unfit for a product that generates a bad outcome. Since \( a = 1 - n \), these comparative statics are reversed for the optimal advice level of the broker. In particular, more advice is optimal when quality is poor (low \( q \)) and products are specialized (low \( \phi \)). Finally, because \( q = \bar{q} \) comes with a cost of \( k_F \) per sale, this cost must be sufficiently small for quality input to be efficient.

2.3 The need for courts

We finish the description of our model by highlighting the fact that courts are crucial for welfare maximization in retail financial markets. Specifically, because courts can enforce contracts that are contingent on the experience of consumers, they alleviate the need for reputation as a driver of incentives for firms and their agents. This is especially important in retail financial markets, as individuals who make use of these markets often do so very sporadically, and it is therefore unlikely that reputation-based equilibria can be easily sustained. For example, the median consumer makes very few mortgage-borrowing or retirement-saving decisions in his or her lifetime.

In the context of our model, the absence of courts automatically leads to the absence of product quality and useful advice. To see this, suppose that \( x_A = x_F = 0 \) and that consumers who are attracted by the broker believe that the firm and broker set \( q = \bar{q} \) and \( a \geq \bar{a}_q \), respectively.\(^{20}\) This means that these consumers are willing to pay as much as

\[ E[U_i(\tilde{x}_i, \tilde{s}_i) | \tilde{s}_i] = [\phi + (1 - \phi)\bar{q}]h - (1 - \phi)(1 - \bar{q}) \frac{1 - a}{2} > 0 \]

for product \( \tilde{s}_i \), and indeed this is the price \( p \) that a profit-maximizing firm would set for its two products. Now consider the incentives of the agent who receives \( w \) per sale. This agent receives \( nw \) from the firm, and does not face any additional cash flows; his incentives are to choose \( n = 1 \) and \( a = 0 \). Knowing this, the firm never sets \( w \) to a strictly positive value, as

\[^{20}A \text{ similar argument applies when consumers believe that } q = 0 \text{ and } a \geq \bar{a}_0.\]
doing so does not affect the agent’s incentives but creates a deadweight loss of \( mw = w \) for the firm; thus, \( w = 0 \). Finally, the firm collects \( p \) for each of the \( n = 1 \) sales, and so its total cash flows consist of revenues of \( p \) and nothing else; that is, the firm has no incentive to incur \( k_F \) for each sale and set \( q \) above zero. In equilibrium, therefore, consumers should expect a complete lack of quality and advice in their valuation of financial products. Since their expected utility from either product is then given by (1), which is negative, they refrain from buying products altogether. That is, the market for retail financial products breaks down.

As we show in the rest of the article, the presence of courts greatly alleviates this holdup problem in quality and advice provision. By facilitating monetary transfers between the firm, agent, and consumers in a state-dependent manner, courts can be used to realign the incentives of the firm and the agent in their choice of \( q \) and \( a \). This realignment is not always perfect, however. When transfers come in the form of penalties imposed by the legal system, we show in Section 3 that realignment to first-best choices of \( q \) and \( a \) is only optimal when courts operate without any frictions (i.e., when \( \lambda = 0 \)). When \( \lambda > 0 \), the legal system faces a trade-off: penalties imposed on the firm and agent create incentives for quality and advice provision, but also introduce welfare-reducing frictions.

Interestingly, as we show in Section 4, it is the combination of courts and third-party regulation that allows for the optimal recovery of surplus through quality and advice. When left to their own device (i.e., without third-party regulation), firms do make use of courts by writing contracts that are contingent on the consumers’ experience with their products, but do so to optimize their own value, not total economic surplus. Specifically, we show that the firm prefers to reduce the agent’s involvement in its relationship with consumers in order to appropriate a larger share and quantity of economic surplus.

### 3. Equilibrium Analysis and Optimal Regulation

In this section, we study the equilibrium behavior of the firm, agent, and consumers, and show how regulation must adjust in order to maximize total welfare. We proceed as follows. First, in Section 3.1, we solve for the breakdown of effort between \( n \) and \( a \) chosen by the agent, given the combined incentives provided by the government and the firm, and given the agent’s beliefs about the firm’s choice of quality. Second, in Section 3.2, we solve for the firm’s optimal choice of quality \( q \), given the incentives provided by the government, and given the firm’s beliefs about the agent’s choice of \( n \) and \( a \). In these first two steps, the price at which the products transact does not play a role, as this price is based on the consumers’ beliefs about the choice of \( a \) and \( q \) by the agent and firm.
Third, in Section 3.3, we solve for the firm’s optimal choice of compensation for the agent. Here, it is the case that the equilibrium price impacts the firm’s decisions. Specifically, a large price potentially creates an incentive for the firm to choose a compensation scheme that implements a regime with low advice and heavy sales. Finally, in Section 3.4, we solve for the legal system that maximizes total welfare. We also show that this legal system implements the first-best scenario when there are no frictions associated with penalties (i.e., when $\lambda = 0$).

### 3.1 The agent’s incentives and effort allocation choices

Suppose that the government sets the legal environment by publicly announcing $L = \{x_F, x_A\}$, with $x_F \geq 0$ and $x_A \geq 0$. After observing this, the firm chooses the compensation $w \geq 0$ that the agent receives for each sale. With this private contract, the firm can partially undo the effect of regulation by appropriately changing the incentives of the agent. Ultimately, the agent receives $w$ for each of the $n$ sales that he intermediates, and faces a penalty of $x_A$ for each of the customers whose utility payoff from their experience with the product is $-\ell$. The firm’s choice of quality, however, cannot be observed by the agent. Anticipating the firm to choose $q \in [0, \bar{q}]$, the agent solves the following problem:

\[
\max_{n, a} n \left[ w - \Pr\{U(\tilde{r}_i, \tilde{s}_i) = -\ell\}x_A \right] = n \left[ w - (1 - \phi)(1 - q) \frac{1 - a}{2} x_A \right]
\]

subject to $n \in [0, 1], \quad a \in [0, 1], \quad n + a \leq 1$.

Since $n + a \leq 1$ always binds, this problem can be rewritten as

\[
\max_{n \in [0, 1]} \left[ w - (1 - \phi)(1 - q) \frac{n}{2} x_A \right],
\]

which yields $^{21}$

\[
n = \frac{w}{(1 - \phi)(1 - q)x_A} \quad \text{and} \quad a = 1 - n.
\]

The comparative statics for $n$ and $a$ trivially go in opposite directions. As the government increases the size of penalties, the agent advises his customers more diligently. In contrast, as the firm increases the sales commission, the agent shifts his priority from advising to selling. That is, by increasing $w$, the firm can partially offset the advising incentives that the government seeks to create via $x_A$. Interestingly, the agent

$^{21}$ Technically speaking, the solution should be written as $n = \min \left\{ 1, \frac{w}{(1 - \phi)(1 - q)x_A} \right\}$ to keep $n$ and $a$ in $[0, 1]$ when $w$ is large. Because the firm never contemplates such large values of $w$, however, we concentrate on the interior solution that will prevail in equilibrium in an effort to ease the exposition.
advises customers less when products are generic. This is because the
agent is protected by a lower likelihood that customers will have a nega-
tive experience, even if they are unfit for the product they buy. So, it
becomes optimal for the agent to advise customers less and sell more
products. The same applies when the agent anticipates the firm to
make high-quality products: as in Groves (1973) and Holmström
(1982), the team structure of the firm and agent allows the agent to
free-ride on the protection that product quality provides.

3.2 The firm’s choice of quality
Just like $x_A$ affects the agent’s incentives to advise consumers, $x_F$
affects the firm’s incentives to create high-quality products (i.e., to choose
$q = \tilde{q}$). Indeed, as we show next, the firm will seek to minimize its legal
liabilities when the impacts of lawsuits ($x_F$) on its bottom line are large
relative to the cost of improving product quality to $q = \tilde{q}$. Given a price $p$
for its products, the firm solves the following problem:

$$
\max_{q \in [0, \tilde{q}]} n \left[ p - w - \Pr(U(\tilde{e}_i, \tilde{s}_j) = -\ell) x_F - k_F \mathbf{1}_{(q = \tilde{q})} \right] = n \left[ p - w - (1 - \phi)(1 - q) \frac{1 - a}{2} x_F - k_F \mathbf{1}_{(q = \tilde{q})} \right].
$$

(14)

Note that, in this maximization problem, $p$ is treated as an exogenous
constant by the firm. Because consumers cannot observe $n, a, q,$ or $w$, the
price $p$ that they are willing to pay for the financial products is obtained
in equilibrium, and not directly affected by the firm’s and the agent’s
decisions.

A simple comparison of (14) with $q = \tilde{q}$ and $q = 0$ shows that $q = \tilde{q}$ is
the firm’s best response to any given $a$ as long as

$$
k_F \leq \frac{\tilde{q}}{2}(1 - \phi)(1 - a)x_F,
$$

(15)

which can be written equivalently as

$$
x_F \geq \frac{2k_F}{\tilde{q}(1 - \phi)(1 - a)}.
$$

(16)

The penalty required for the firm to produce a high-quality product is
increasing in $k_F, \phi,$ and $a$. Intuitively, the firm will save the cost of
improving quality when this cost is large (i.e., when $k_F$ is large) and when
few consumers are likely to suffer from badly performing products (i.e.,
when $(1 - \phi)$ is small). The fact that the penalty threshold for $q = \tilde{q}$ must
increase with $a$ comes from free-riding (as in the moral-hazard-in-teams
problem of Groves 1973 and Holmström 1982): knowing that the agent
responsibly advises customers based on their needs (i.e., when $a$ is large),
the firm faces diminished incentives to augment quality, as the effect this has on expected fines is then small. Finally, for a given cost \( k_F \) and penalty size \( x_F \), the firm is more likely to choose \( q = \tilde{q} \) when quality can be improved significantly (i.e., when \( \tilde{q} \) is large). Alternatively, noting that \( n = 1 - a \), we can rewrite (16) as \( x_F \geq \frac{2k_F}{q(1 - \phi)w} \): the firm is more prone to choosing \( q = \tilde{q} \) when it expects a large number of sales, as this can result in proportionately more lawsuits.

Two equilibria for the agent’s choice of \( n \) and the firm’s choice of \( q \) are possible: one with \( q = \tilde{q} \), and one with \( q = 0 \). We refer to the former as the \textit{responsible} equilibrium and to the latter as the \textit{irresponsible} equilibrium. For the responsible equilibrium to obtain, it must be the case that the agent expects \( q = \tilde{q} \) and that it is then optimal for the firm to choose \( q = \tilde{q} \). That is, (16) must hold with \( 1 - a = n = \frac{w}{(1 - \phi)(1 - q)x_A} \), as derived in (13). This condition simplifies to

\[
x_F \geq \frac{2k_F(1 - \tilde{q})x_A}{\tilde{q}w},
\]

and the irresponsible equilibrium prevails when it does not hold.\(^{22}\) This expression highlights some of the tensions that exist as a result of the team structure between the firm and its agent. First, keeping \( w \) fixed, an increase in \( x_A \) requires a commensurate increase in \( x_F \) in order to preserve the responsible equilibrium. Second, the responsible equilibrium requires larger penalties when the amount of quality that the firm can inject into its products is limited (i.e., \( \tilde{q} \) is small). Third, because larger commissions create an incentive for the agent to sell more and advise less, smaller penalties are necessary in order to motivate the firm to act responsibly, as the firm seeks to protect itself from more potential lawsuits.

\subsection*{3.3 The firm’s choice of compensation contract}

The firm’s incentives in setting the agent’s compensation are affected by the price \( p \) that consumers are willing to pay for the financial product. Indeed, as we show next, it is tempting for the firm to motivate the agent to sell more and advise less when \( p \) is large. That is, as \( p \) increases, the extra revenues from irresponsible sales that are made with little advice more than offset the extra fines that these sales create.

To see this, let us consider the maximization problem that the firm faces at \( t = 2 \) when it sets the size of the agent’s commission for each sale. At that time, knowing (13), the firm can use \( w \) to effectively motivate the agent to choose any \( n \in [0,1] \), with a corresponding advice level of

\(^{22}\) To be perfectly precise, it can be shown that both equilibria can be sustained for \( w \in \left[ \frac{2k_F(1 - \tilde{q})x_A}{\tilde{q}w}, \frac{2k_Fx_A}{\tilde{q}w} \right] \), but the responsible equilibrium Pareto dominates the irresponsible equilibrium and so we assume that it prevails.
\( a = 1 - n \). Of course, as Lemma 2 shows, this choice of \( w \) will be different in the two potential equilibria.

**Lemma 2**

Suppose that the price of the firm’s products is set at \( p \) and that the consumers who are attracted by the agent are willing to pay this price. In the equilibrium with \( q \in \{0, \bar{q}\} \), the firm sets the per-sale commission of the agent to

\[
\begin{align*}
\nonumber w &= \frac{(p - k_F \mathbf{1}_{\{q=\bar{q}\}}) x_A}{2x_A + x_F} \\
\text{and, as a result, the agent chooses} \\
\nonumber n &= \frac{p - k_F \mathbf{1}_{\{q=\bar{q}\}}}{(1 - \phi)(1 - q)(2x_A + x_F)} \quad \text{and} \quad a = 1 - n.
\end{align*}
\]

As seen in (18), the firm offers the agent a larger sales commission when the price it charges for its products is large. In turn, (19) shows that this motivates the agent to shift some of his effort capital from advising to selling. Interestingly, \( w \) is increasing in \( x_A \) and decreasing in \( x_F \). When the agent is hit with large penalties for mis-selling, the firm must reward him more for each risky sale. In contrast, when the firm’s penalties are large, it is optimal for the firm to provide the agent with weaker selling incentives and stronger advising incentives. Ultimately, for a given price \( p \), the firm implements a more dependable advice regime when the penalties \( x_A \) and \( x_F \) set by the government are large (i.e., the equilibrium advice level \( a \) is increasing in both \( x_A \) and \( x_F \)).

We can find the maximum price that the customers attracted by the agent are willing to pay for the firm’s products in this economy. These customers observe \( \mathcal{L} = \{x_F, x_A\} \) and can anticipate the agent’s and firm’s equilibrium choices of \( a \) and \( q \) based on (16) and (19). Specifically, upon being recommended product \( \tilde{s}_i = t \), customer \( i \)’s expected utility from the purchase is given by

\[
\begin{align*}
\nonumber E[U(\tilde{t}_i, t) \mid \tilde{s}_i = t] &= \Pr(\tilde{r}_i^j = G) h + \Pr(\tilde{r}_i^j = B) \Pr(\tilde{t}_i \neq t \mid \tilde{s}_i = t) \\
\nonumber &= \left[ -\ell + (1 - \lambda)(x_A + x_F) \right] \\
\nonumber &= [\phi + (1 - \phi)q] h - (1 - \phi)(1 - q) \frac{1 - a}{2} [\ell - (1 - \lambda)] \quad (x_A + x_F)]. \\
\end{align*}
\]

\*23 As Lemma 3 will verify, it is always the case that \( p > k_F \) in the responsible equilibrium, so that \( w \) is always greater than zero.
Assuming that the equilibrium with \( q \in [0, \tilde{q}] \) prevails and using (19) for \( 1-a \), we can write this last expression as a function of \( p \). The quantity obtained must also be equal to the equilibrium price, as the firm maximizes profits by setting \( p \) to the largest price that customers are willing to pay. This search for a fixed point in \( p \) is the object of Lemma 3.

**Lemma 3**

*In the equilibrium with \( q \in [0, \tilde{q}] \), the price of the firm’s products is given by*

\[
p = \frac{2h(2x_a + x_f)[\phi + (1 - \phi)q] + k_F 1_{(q=\tilde{q})} [\ell - (1 - \lambda)(x_a + x_f)]}{\ell + (3 + \lambda)x_a + (1 + \lambda)x_f} \equiv P(q) > k_F 1_{(q=\tilde{q})}.
\]

It is straightforward to verify that \( P(\tilde{q}) \) and \( P(0) \) are both increasing in \( h, \phi, \) and \( \tilde{q} \), and decreasing in \( \ell \) and \( \lambda \).\(^{24}\) This is intuitive. The good outcomes are more frequent and yield more utility to consumers when \( h, \phi, \) and \( q \) are large; as such, they are willing to pay more for the products. In contrast, consumers fear large negative utility shocks when \( \ell \) is large and benefit less from regulated penalties imposed on the firm and agent when \( \lambda \) is large; as a result, they pay less for the products.

The impact of penalties on \( P(\tilde{q}) \) and \( P(0) \) is more subtle, as the penalties affect not only what wronged consumers receive but also the direct incentives they have on the firm and agent, as well as the incentives that the firm creates for the agent through its choice of \( w \). In particular, an increase in \( x_A \) leads to two opposite forces on \( n \) (and on \( a = 1 - n \)). The direct effect is to make the agent shift his effort from selling to advising in order to reduce his expected penalties. At the same time, the increase in \( x_A \) leads the firm to increase \( w \) (see (18) in Lemma 2), and this motivates the agent to sell more and advise less. It can be shown that, when \( x_F \) is sufficiently large, this last force dominates: although wronged customers receive more from the agent, they cannot rely as much on his advice at the time they buy the product. This is why the equilibrium price is nonmonotonic in \( x_A \). On the other hand, an increase in \( x_F \) leads the firm to reduce \( w \), which in turn leads to an increased advice level, which also

\(^{24}\) Of course, \( P(0) \) is not affected by changes in \( \tilde{q} \). The two less trivial comparative statics for \( P(\tilde{q}) \) are with respect to \( \ell \) and \( \lambda \). To verify the former, notice that

\[
\frac{\partial P(\tilde{q})}{\partial \ell} = k_F \frac{\ell + (3 + \lambda)x_A + (1 + \lambda)x_f - 2h(2x_A + x_f)[\phi + (1 - \phi)\tilde{q}] - k_F [\ell - (1 - \lambda)(x_A + x_f)]}{[\ell + (3 + \lambda)x_A + (1 + \lambda)x_f]^2}
\]

\[
= \frac{-2(2x_A + x_f)[(\ell)(\phi + (1 - \phi)\tilde{q}) - k_F]}{[\ell + (3 + \lambda)x_A + (1 + \lambda)x_f]^2},
\]

which is smaller than zero by (a2). The latter can be verified similarly.
benefits consumers. Thus, the equilibrium price is monotonically increasing in $x_F$.\footnote{To see all this, first notice that we can write $P(q) = \frac{2h[\phi + (1 - \phi)q]}{1 - \phi}(1 - q)[\ell + (3 + \lambda)x_A + (1 + \lambda)x_F]$, where $L = \frac{\ell - (1 - \lambda)(x_A + x_F)}{2x_A + x_F}$. It is then easy to show that $\frac{\partial P(q)}{\partial L} = \frac{-2}{(2 + L)}\left(h[\phi + (1 - \phi)q] - k_F1_{q=q}\right) < 0$, where the inequality is implied by (a2). The comparative statics with respect to $x_A$ and $x_F$ follow from the fact that $\frac{\partial a}{\partial x_A} < 0$ if and only if $x_F < \frac{\partial a}{\partial x_F}$.}

For further insight about how this equilibrium price interacts with the agent’s effort allocation choices, let us insert $p = P(q)$ from (21) in (19):

$$n = \frac{2h[\phi + (1 - \phi)q] - 2k_F1_{q=q}}{(1 - \phi)(1 - q)[\ell + (3 + \lambda)x_A + (1 + \lambda)x_F]} \equiv N(q).$$

It is straightforward to verify that $N(\bar{q})$ is increasing in $h$, $\phi$, and $\bar{q}$, and decreasing in $\ell$, $\lambda$, $x_A$, and $x_F$ (and similarly for $N(0)$). Since $a = 1 - n$, it is therefore the case that in equilibrium the agent is more dedicated to advising than selling when products are specialized (small $\phi$) and generate small gains (small $h$) but potentially great losses (large $\ell$). It is also the case that quality and advice are substitutes in equilibrium: an increase in $\bar{q}$ leads to a decrease in $a$. Finally, penalties have the desired effect in that they curb the moral hazard incentives to sell without guidance. That is, increases in $x_A$ and $x_F$ lead to fewer sales but to better matches of consumers with products.

### 3.4 The government’s choice of regulation

We now analyze consumer protection law in this market. Specifically, we derive and characterize the legal rules that maximize welfare in the economy.

Since prices, wages, and penalties are transfers among market participants, total welfare equals the utility gains and losses that consumers experience minus the deadweight losses associated with penalties and the firm’s cost to improve quality in the responsible equilibrium. Thus, total welfare differs from $\mathcal{W}_0$ [introduced earlier in (6)] by

$$n \Pr(U(\bar{r}, \bar{s}) = -\ell)\lambda(x_A + x_F) = n(1 - \phi)(1 - q)\frac{1 - a}{2}\lambda(x_A + x_F),$$

and so we write it as

$$\mathcal{W}_L = \mathcal{W}_0 - n(1 - \phi)(1 - q)\frac{1 - a}{2}\lambda(x_A + x_F)$$

$$= n\left(\phi + (1 - \phi)q\right)h - (1 - \phi)(1 - q)\frac{1 - a}{2}\left[\ell + \lambda(x_A + x_F)\right] - k_F1_{q=q}.\right)$$

(23)
This is the quantity that the government seeks to maximize by setting the law \( L = \{x_F, x_A\} \), which affects the broker’s choice of \( n \) and \( a \), as well as the firm’s choice of \( q \) and \( w \), and in turn the consumer surplus that gets realized. The full solution to the government’s problem is characterized in Proposition 2.

**Proposition 2**

(i) If \( k_F < \hat{k}_\lambda \) (with \( \hat{k}_\lambda > 0 \) derived in the proof), then the government maximizes \( \mathcal{W}_\lambda \) by setting a law that implements a responsible equilibrium (\( q = \bar{q} \)) with

\[
\begin{align*}
\frac{3h\bar{q}[\phi + (1 - \phi)\bar{q}] - k_F[3\bar{q} + 2\lambda(1 - \bar{q})]}{3(1 - \phi)\bar{q}(1 - \bar{q})} &\in (0, 1 - \bar{a}_0) \\
\end{align*}
\]

and \( a = 1 - n \). The law is given by

\[
\begin{align*}
x_F &= \frac{2k_F}{\bar{q}(1 - \phi)n} > 0, \quad \text{and} \\
x_A &= \frac{1}{3 + \lambda} \left( \frac{2h[\phi + (1 - \phi)\bar{q}] - 2k_F}{(1 - \phi)(1 - \bar{q})n} - (1 + \lambda)\bar{x}_F - \ell \right) > 0. \\
\end{align*}
\]

(ii) If \( k_F \in [\hat{k}_\lambda, \hat{k}_\lambda] \) (with \( \hat{k}_\lambda \geq \hat{k}_\lambda \) derived in the proof), then the government maximizes \( \mathcal{W}_\lambda \) by setting a law that implements a responsible equilibrium (\( q = \bar{q} \)) with

\[
\frac{2h\bar{q}[\phi + (1 - \phi)\bar{q}] - 2k_F[1 + \lambda(1 - \bar{q})]}{\ell(1 - \phi)\bar{q}(1 - \bar{q})} \in (0, 1 - \bar{a}_0) 
\]

and \( a = 1 - n \). The law is given by (25) and \( x_A = 0 \).

(iii) If \( k_F \geq \hat{k}_\lambda \), then the government maximizes \( \mathcal{W}_\lambda \) by setting a law that implements an irresponsible equilibrium (\( q = 0 \)) with

\[
\frac{h\phi}{\ell(1 - \phi)} \in (0, 1 - \bar{a}_0) \quad \text{and} \quad a = 1 - n. \quad \text{The law is given by} \quad x_F = 0 \quad \text{and} \quad x_A = \frac{h}{3 + \lambda}. 
\]

When the cost of quality \( k_F \) is small, the government finds it optimal to increase the firm’s penalties sufficiently in order to implement the responsible equilibrium (with \( q = \bar{q} \)). As \( k_F \) increases, the government must increase \( x_F \) in order to sustain this equilibrium, as the firm then requires a stronger incentive to increase quality. At the same time, to limit the size of the deadweight losses associated with penalties, the government

\[\text{Because } x_A = 0 \text{ in equilibrium, it is tempting to conclude that the agent has no incentive to advise customers (and so picks } a = 0 \text{ and } n = 1), \text{ as he never owes them anything ex post. This intuition is incorrect. Specifically, we know from (13) that the ratio of } x_A \text{ to } w \text{ determines the agent’s allocation of effort: a reduction in } x_A \text{ leads to more sales and less advice, whereas a reduction in } w \text{ leads to fewer sales and more advice. Thus, as the government reduces } x_A, \text{ the firm simultaneously and proportionately reduces } w \text{ to implement any } n \text{ that it considers optimal.} \]
reduces the broker’s penalties, $x_A$, and eventually reaches a situation [in part (ii)] where it is no longer optimal to penalize the broker. However, when $k_F$ becomes too large, the government gives up on motivating the firm to improve the quality of its products, and instead relies exclusively on the broker’s advice, motivated through $x_A > 0$, to maximize welfare.

Before we look at the comparative statics for $n$, $x_F$, and $x_A$, it is useful to analyze the equilibrium from part (i) of Proposition 2 (i.e., the equilibrium with $q = \bar{q}$ and $x_A > 0$) in the absence of frictions (i.e., with $\lambda = 0$). In this case, (24) reduces to

$$n = \frac{h[\phi + (1 - \phi)\bar{q}] - k_F}{\ell(1 - \phi)(1 - \bar{q})},$$

as in (9). That is, with a careful choice of penalties, the government is able to recreate the first-best scenario of Proposition 1. When $\lambda > 0$, this is no longer possible, as any penalty imposed on the firm or the agent comes with a deadweight loss. As such, the government must balance these losses in its choice of $x_F$ and $x_A$, and the resulting equilibrium departs from first-best.

Proposition 3 characterizes the number of sales and the agent’s advice level in equilibrium. It also summarizes the properties of the penalties imposed on the firm and agent in equilibrium. Specifically, it looks at the sum of $x_A$ and $x_F$ that wronged consumers receive, and at the ratio of $x_A$ to $x_A + x_F$, which measures the share of the blame that the law imposes on the agent. The proposition concentrates on small values of $k_F$ to avoid the corner solutions ($x_A = 0$ or $x_F = 0$) that can result otherwise.

**Proposition 3**

*Suppose that $k_F < \hat{k}_\lambda$ (so that $q = \bar{q}$, $x_F > 0$, and $x_A > 0$ prevail in equilibrium).*

(i) In equilibrium, the number of sales (the agent’s advice level) is smaller (larger) than in first-best, increasing (decreasing) in $\bar{q}$, $\phi$, and $h$, and decreasing (increasing) in $k_F$, $\ell$, and $\lambda$.

(ii) The total payments $x_A + x_F$ that are made to wronged consumers are decreasing in $\bar{q}$, $\phi$, and $h$, and increasing in $k_F$ and $\ell$.

(iii) The agent’s portion $\frac{x_A}{x_A + x_F}$ of these penalties is increasing in $\bar{q}$, $\phi$, and $h$, decreasing in $k_F$, and unaffected by changes in $\ell$.

From part (i) of Proposition 3, we see that the penalties chosen by the government increase the level $a$ of advice above first-best. That is, the government finds it optimal to slow down the pace at which sales are made (i.e., $n$ is lower than in first-best) and implements a regime that emphasizes consumer protection. How this is achieved is described by parts (ii) and (iii) of the proposition.
When ex ante the consumers’ experience with the product is likely to be good (i.e., when $\phi$ and $h$ are large), the government finds it optimal to reduce the magnitude of its costly interventions and to let the firm and agent distribute the product to more people, as advice is then not that crucial. Similarly, when the firm’s product quality can be greatly improved at a low cost (i.e., when $\hat{q}$ is large and $k_F$ is small), the government plays a more limited role in the economy. It is when consumer losses are potentially large (i.e., when $\ell$ is large) that the government increases its oversight of retail financial markets by raising the awards that go to wronged consumers. This increased vigilance is worth the additional costs implicit in the lower number of sales that take place and in the frictional losses associated with the use of courts.

Interestingly, increases in the size of the government’s interventions also come with relatively more responsibility on the firm’s part than on the agent’s part. That is, although the agent produces the advice, the government internalizes the incentives of the firm to lower the agent’s compensation in order for him to increase sales and lower advice.

4. The Suboptimality of Self-regulation

So far, we have assumed that only the government makes use of the court system that is in place. Since courts serve to enforce contracts that are contingent on negative outcomes for consumers, there is nothing preventing firms from writing such contracts. In this section, we investigate this possibility. Specifically, we analyze a situation in which the firm, as opposed to the government, sets the size of the payments, $x_F$ and $x_A$, that the firm and agent make to consumers who experience bad outcomes with the firm’s products. As we show, the team problem that constrains the government’s maximization of welfare in Section 3.4 now pushes the firm to choose contractual terms that fail to maximize economic surplus and instead tilt economic outcomes in its favor.

With the use of courts, the firm’s problem is to maximize expected profits by choosing $x_F$ and $x_A$, as well as $p$, as part of the terms of exchange with consumers. When setting these quantities at the outset, the firm internalizes the game that it and the agent subsequently play, like

---

Note that in principle the firm could also make the agent’s compensation contract contingent on the utility outcomes of consumers. Setting the transfers from the firm and agent to wronged consumers (i.e., $x_F$ and $x_A$) is sufficient, however, as the firm could equivalently set $x_A$ as the transfer from the agent to the firm and $x_A + x_F$ as the transfer from the firm to a consumer $i$ whose payoff turns out to be $-\ell$. 
the government did in Section 3.4. Specifically, the firm seeks to solve the following maximization problem:

\[
\max \ n \left[p - w - \Pr[U(\tilde{r}, \tilde{s}) = -\ell] x_F - k_F I_{q=\tilde{q}}\right]
\]

\[
= n \left[p - w - (1 - \phi)(1 - q) \frac{1 - a}{2} x_F - k_F I_{q=\tilde{q}}\right],
\]

subject to the equilibrium choices of the firm and agent that come with a given pair \(\{x_F, x_A\}\) in Sections 3.1–3.3. The solution to this problem is the object of Proposition 4.

**Proposition 4**

(i) If \(k_F < \tilde{k}_\lambda\) (with \(\tilde{k}_\lambda > 0\) derived in the proof), then the firm maximizes expected profits by implementing \(q = \tilde{q}\),

\[
n = \frac{2h\tilde{q}[\phi + (1 - \phi)\tilde{q}] + k_F[1 - 3\tilde{q} - \lambda(1 - \tilde{q})]}{2\ell(1 - \phi)\tilde{q}(1 - \tilde{q})} \in (0, 1),
\]

and \(a = 1 - n\). It does so by setting

\[
x_F = \frac{2k_F}{\tilde{q}(1 - \phi)n} > 0,
\]

and

\[
x_A = \frac{1}{3 + \lambda} \left(\frac{2h[\phi + (1 - \phi)\tilde{q}] - 2k_F}{(1 - \phi)(1 - \tilde{q})n} - (1 + \lambda)x_F - \ell\right) > 0.
\]

(ii) If \(k_F \in [\tilde{k}_\lambda, \tilde{k}_\lambda]\) (with \(\tilde{k}_\lambda > \tilde{k}_\lambda\) derived in the proof), then the firm maximizes expected profits by implementing \(q = \tilde{q}\),

\[
n = \frac{2h\tilde{q}[\phi + (1 - \phi)\tilde{q}] - 2k_F[1 + \lambda(1 - \tilde{q})]}{\ell(1 - \phi)\tilde{q}(1 - \tilde{q})} \in (0, 1),
\]

and \(a = 1 - n\). It does so by setting \(x_F\) as in (31) and \(x_A = 0\).

(iii) If \(k_F \geq \tilde{k}_\lambda\), then the firm maximizes expected profits by implementing \(q = 0\), \(n = \frac{h\lambda}{\ell(1 - \phi)}\), and \(a = 1 - n\). It does so by setting \(x_F = 0\) and \(x_A = \frac{\ell}{3+\lambda}\).

Like the government, the firm finds it optimal to implement a responsible regime when the cost of doing so is sufficiently small as, otherwise, it is cheaper to rely exclusively on the agent for reducing the frequency of product-consumer mismatches. However, as our next proposition shows, the penalties \(x_F\) and \(x_A\) that the firm selects for low values of \(k_F\) are smaller than those chosen by the government.\(^28\) That is, the firm prefers a laissez-faire approach that increases sales volume, at the expense of consumer guidance.

\(^{28}\) A comparison of parts (ii) and (iii) between Proposition 2 and Proposition 4 reveals that the government and firm eventually reach similar policies as \(k_F\) gets larger.
Before we formally establish these results and more generally characterize the properties of the solution derived in Proposition 4, let us contrast the firm’s problem with that of the government in Section 3.4 by inserting $p$ from (20) in (29). The firm’s problem can then be rewritten as

$$\max_{x_F, x_A} \left\{ [\phi + (1 - \phi)q]h - (1 - \phi)(1 - q) \frac{1-a}{2} [\ell - (1 - \lambda)(x_A + x_F)] - w - (1 - \phi)(1 - q) \frac{1-a}{2} x_F - k_F 1_{(q=q)} \right\}$$

$$= n \left\{ (1 - \phi)(1 - q) \frac{1-a}{2} x_A - w + [\phi + (1 - \phi)q]h - (1 - \phi)(1 - q) \frac{1-a}{2} [\ell + \lambda(x_A + x_F)] - k_F 1_{(q=q)} \right\}$$

$$= W_\lambda - \frac{1}{2} nw,$$

where (13) and (23) were used to obtain this last line. This expression highlights the effect of the team arrangement of the firm and agent on the firm’s problem. If the firm were to choose the same $x_F$ and $x_A$ as the government, the same economic surplus $W_\lambda$ would get realized but the agent would then capture half of $w$ for each sale that is made. This is due to the fact that, when $x_A > 0$, the firm must motivate the agent to sell the product (i.e., to pick $n > 0$) by increasing his commission $w$ for each sale. This leaves the agent with an opportunity to capture some of the economic surplus by optimizing his choices of $n$ and $a$. Specifically, we can see from the agent’s maximization problem and solution in (12) and (13) that the agent chooses $n$ so that his expected profits are $\frac{1}{2} nw$. Effectively, therefore, the firm bears the full cost $nw$ of creating incentives for the agent, but its benefit is reduced by half of this quantity.

Since the firm’s optimal choice of $w$ is increasing in $x_A$ [see (18) in Lemma 2], the firm can (and does) reduce the agent’s share of the economic surplus by decreasing both $x_A$ and $w$. It is also the case that $k_\lambda < \bar{k}_\lambda$: the firm fully eliminates agent penalties for lower values of $k_F$ than the government. As our final result shows, these distortions in the firm’s problem lead it to implement a regime of heavy sales and low advice, reducing the potency of the court system and in turn the price that consumers are willing to pay for the firm’s products.

**Proposition 5**

*Suppose that $k_F < \bar{k}_\lambda$, so that $q = \bar{q}$, $x_A > 0$, and $x_F > 0$ prevail, whether $x_F$ and $x_A$ are set by the firm or by the government.*

(i) In equilibrium, the advice level $a$, the firm’s and agent’s penalties $x_F$ and $x_A$, the agent’s per-sale commission $w$, and the equilibrium price $p$ are
all smaller when the firm self-regulates than when the government sets the law; the opposite is true for the number of sales \( n \).

(ii) The firm’s self-regulation keeps the advice level \( a \) (the number of sales \( n \)) strictly below (above) its first-best level, even when \( \lambda = 0 \).

Proposition 5 highlights the role of regulation in retail financial markets. When product sales and customer advice are intermediated, the team incentives that endogenously arise between the producer and the broker make it impossible for the firm to maximize economic surplus while self-regulating. Instead, the firm has an incentive to reduce the agent’s share of the surplus by reducing his compensation and his advising incentives, preferring instead that the agent concentrates his efforts on selling more of its products. The larger number of sales reduces the size of the penalties that the firm must impose on itself to signal its commitment to quality, as the firm then potentially faces more claims from mismatched consumers. In turn, both the reduction in transfers that customers can expect when they are mismatched with a product and the lower advice level they receive from the broker reduce their willingness to pay for the product.

Although more customers ultimately benefit (ex ante) from the firm’s products, realized consumer surplus (and thus total welfare) is not as large as when the government controls \( x_f \) and \( x_A \) via regulation. Indeed, the fact that the firm prefers a larger fraction of a smaller pie has a negative overall effect on consumers. In this light, the advantage of having the government set the law becomes even clearer: its optimal use of penalties restores consumer confidence, increases prices, and increases total welfare.

Finally, as the second part of Proposition 5 shows, the firm always prefers to over-sell and under-advice relative to first-best. This is in stark contrast to our earlier result that the government maximizes total welfare via a regime of under-selling and over-advising relative to first-best. Moreover, whereas the government’s regulation approaches first-best as frictions become smaller (i.e., as \( \lambda \) goes to zero), the equilibrium is bounded away from first-best when the firm self-regulates. Again, this is due to the fact that the firm does not capture the entire economic surplus in its bilateral arrangement with the agent. Thus, when left to self-regulate, the firm loosens the penalties even if their effectiveness is not weakened by frictions.

5. Concluding Remarks

Protecting consumers in financial markets who are “unable to fend for themselves” is not only an important duty of the law, but also an important driver of participation in the market and economic growth. In this article, we characterize the legal rules that maximize social welfare
in markets in which producers of financial markets outsource their advice services. The model we analyze is one of bilateral hidden action: firms choose the quality of the goods they produce and brokers advise consumers when they make their purchases. Without the law, neither party can commit to acting in the best interest of consumers, and none of the economic surplus that markets can potentially generate is actually realized. With the law, the two parties tend to free-ride on each other’s effort provision: as the firm commits to higher quality, the broker has a lower incentive to give advice, and vice versa.

We show that the optimal law sets the total penalty level and the split of the blame based on the product characteristics and the relative ability of the firm to improve product quality. The law tends to be stricter and attribute more of the blame to the agent when the products are generic, when the firm can easily control quality, and when products have a relatively large upside.

In all cases, however, government regulation always implements a regime of care in which transactions are fewer but better advised than in first-best. In contrast, when the firm is allowed to self-regulate, it favors a high-volume regime with less advising care by the broker, as this tends to shift more of the economic surplus from the broker to the firm even though it also reduces social welfare.

Appendix

Proof of Lemma 1
Using Bayes’ rule, we have

$$\Pr(\tilde{t}_i = t | \tilde{s}_i = t) = \frac{\Pr(\tilde{t}_i = t | \tilde{s}_i = t) \Pr(\tilde{s}_i = t)}{\Pr(\tilde{s}_i = t)} = \frac{[a + \frac{1}{2}a^2]}{2} = \frac{1 + a}{2}.$$ 

This completes the proof. ■

Proof of Proposition 1
The first-order condition with respect to \(n\) yields (9), and it is straightforward to verify that the second-order condition is satisfied. With this \(n\), the total welfare in (8) is equal to

$$W_0(q) \equiv \frac{\left[h(\phi + (1 - \phi)q) - k_F I_{(q=\tilde{q})}\right]^2}{2\ell(1 - \phi)(1 - q)}.$$ 

It is optimal to set \(q = \tilde{q}\) if and only if \(W_0(\tilde{q}) \geq W_0(0)\). It is straightforward to verify that this condition is equivalent to (11). ■

Proof of Lemma 2
The firm sets \(w\) anticipating the game that it and the agent subsequently play. That is, it chooses \(w \geq 0\) in order to maximize its expected profits in (14),

$$n \left[ p - w - (1 - \phi)(1 - q) \frac{1 - a}{2} x_F - k_F I_{(q=\tilde{q})} \right].$$
subject to (13). By inserting (13) into the firm’s expected profits, we can rewrite this as follows:

$$\max_n n \left[ p - n(1 - \phi)(1 - q)x_A - \frac{n}{2} (1 - \phi)(1 - q)x_F - k_F 1_{1 q = \bar{q}} \right].$$

(A1)

This expression is maximized at $n$ given by (19). We can then use (13) to obtain (18).

**Proof of Lemma 3**

After inserting (19) for $n = 1 - a$ in (20) and setting the result equal to $p$, we have

$$p = \left[ \phi + (1 - \phi)q \right] \hat{h} - \frac{p - k_F 1_{1 q = \bar{q}}}{2(2x_A + x_F)} \left[ \ell - (1 - \lambda)(x_A + x_F) \right].$$

Solving for $p$, we find (21). Clearly,

$$P(0) = \frac{2h(2x_A + x_F)\phi}{\ell + (3 + \lambda)x_A + (1 + \lambda)x_F} > 0.$$

Also, we have

$$P(\bar{q}) - k_F = \frac{2h(2x_A + x_F)[\phi + (1 - \phi)\bar{q}] + k_F[\ell - (1 - \lambda)(x_A + x_F) - \ell - (3 + \lambda)x_A - (1 + \lambda)x_F]}{\ell + (3 + \lambda)x_A + (1 + \lambda)x_F} = \frac{2(2x_A + x_F)[h(\phi + (1 - \phi)\bar{q}) - k_F]}{\ell + (3 + \lambda)x_A + (1 + \lambda)x_F} > 0,$$

where the inequality results from (a2). Thus, $P(\bar{q}) > k_F$.

**Proof of Proposition 2**

The government must choose $x_F \geq 0$ and $x_A \geq 0$ in order to maximize (23), subject to the firm’s equilibrium choices of $q$ and $w$ (in Sections 3.2 and 3.3) and the agent’s equilibrium choices of $n$ and $a$ (in Section 3.1). Let us first conjecture a responsible equilibrium in which the firm chooses $q = \bar{q}$. We can use (22) with $q = \bar{q}$ to write $x_A$ as a function of $n$ and $x_F$:

$$x_A = \frac{1}{3 + \lambda} \left( \frac{2h[\phi + (1 - \phi)\bar{q}] - 2k_F}{(1 - \phi)(1 - \bar{q})n} - (1 + \lambda)x_F - \ell \right).$$

(A2)

This is (26). After replacing $x_A$ in (23) by this last expression and simplifying, the government’s problem becomes

$$\max_{n, x_F} \frac{n}{3 + \lambda} \left\{ 3h[\phi + (1 - \phi)\bar{q}] - \frac{3n}{2} (1 - \phi)(1 - \bar{q})\ell - n(1 - \phi)(1 - \bar{q})\lambda x_F - 3k_F \right\}. \tag{A3}$$

For the responsible equilibrium to obtain, it must be the case that $x_F$ satisfies (16). Since $n = 1 - a$ and since it can never be optimal for the government to set $x_F$ greater than necessary (as doing so does not affect the firm’s choice of $q = \bar{q}$ and results in more deadweight losses through $\lambda x_F$), $x_F$ must be given by (25).

After we insert (25) in (A3), the government’s maximization problem simplifies to

$$\max_n \frac{n}{3 + \lambda} \left\{ 3h[\phi + (1 - \phi)\bar{q}] - \frac{3n}{2} (1 - \phi)(1 - \bar{q})\ell - \frac{k_F}{\bar{q}}[3\bar{q} + 2\lambda(1 - \bar{q})] \right\}. \tag{A4}$$

29 Mathematically, since (A3) is strictly decreasing in $x_F$, it must be the case that $x_F$ is either the smallest possible value that makes the firm choose $q = \bar{q}$, or $x_F$ is set equal to zero in which case the firm chooses $q = 0$ (as in part (iii) of the proposition).
The first-order condition with respect to $n$ yields (24). This $n$ is positive since
\[
3h\phi(1 - \phi)q - k_F[3\phi + 2\lambda(1 - \phi)] \geq 3h\phi(1 - \phi)q - k_F[2\phi + 1] > h\phi(3\phi - 1)(1 - \phi) > 0,
\]
where the first inequality follows from $\lambda \leq \frac{1}{2}$, the second from (a2), and the last inequality from $\phi \geq \frac{1}{2}$. Also, to show that (24) is smaller than $1 - \tilde{a}_q$ [with $\tilde{a}_q$ defined in (7)], notice that this is equivalent to showing that
\[
3h\phi(1 - \phi)q - k_F[3\phi + 2\lambda(1 - \phi)] < 6h\phi(1 - \phi)q - 6k_F\phi q,
\]
which is itself equivalent to
\[
-3h\phi(1 - \phi)q + k_F[3\phi + 2\lambda(1 - \phi)] < 0.
\] (A5)

If $3\phi - 2\lambda(1 - \phi) < 0$, this clearly holds. Otherwise, (a2) implies
\[
-3h\phi(1 - \phi)q + k_F[3\phi - 2\lambda(1 - \phi)] < -3h\phi(1 - \phi)q + h\phi[3\phi - 2\lambda(1 - \phi)]
\]
\[
= -h\phi[3\phi(1 - \phi) + 2\lambda(1 - \phi)] < 0.
\]

Thus, (A5) holds, and we have $n < 1 - \tilde{a}_q).

Given that $n > 0$, it is clearly the case that (25) is positive. We next verify that (26) is also positive for $k_F$ sufficiently small. Let us first insert (25) in (26) and simplify:
\[
x_A = \frac{1}{3 + \lambda} \left( \frac{2h\phi (1 - \phi)q - 2k_F[1 + \lambda(1 - \phi)]}{(1 - \phi)q(1 - \phi)n} - \ell \right).
\] (A6)

Now, let us insert (24) for $n$ in this last expression:
\[
x_A = \frac{\ell}{3 + \lambda} \left( \frac{6h\phi (1 - \phi)q - 6k_F[1 + \lambda(1 - \phi)]}{3h\phi (1 - \phi)q - k_F[3\phi + 2\lambda(1 - \phi)] - 1} \right)
\]
\[
= \frac{\ell}{3 + \lambda} \left( \frac{3h\phi (1 - \phi)q - k_F[3\phi + 2\lambda(1 - \phi)]}{3h\phi (1 - \phi)q - k_F[3\phi + (3 + 4\lambda)(1 - \phi)]} - 1 \right)
\] (A7)
\[
= \frac{\ell}{3 + \lambda} \left( 1 - \frac{2k_F(3 + \lambda)(1 - \phi)}{3h\phi (1 - \phi)q - k_F[3\phi + 2\lambda(1 - \phi)]} \right).
\]

This last quantity is positive if and only if
\[
3h\phi (1 - \phi)q - k_F[3\phi + 2\lambda(1 - \phi)] > 2k_F(3 + \lambda)(1 - \phi),
\]
which is equivalent to
\[
3h\phi (1 - \phi)q > k_F[2(3 + \lambda)(1 - \phi) + 3\phi + 2\lambda(1 - \phi)],
\]
and in turn to
\[
k_F < \frac{3h\phi (1 - \phi)q}{2k_F[2(3 + \lambda)(1 - \phi) + 3\phi + 2\lambda(1 - \phi)]} = \hat{k}_F \in (0, h\phi).
\] (A8)

The government can also implement the responsible equilibrium for larger values of $k_F$ by setting $x_A$ equal to zero and $x_F$ sufficiently large for the firm to choose $q = \bar{q}$, i.e., $x_F$ as in (25). In this case, the government effectively loses its ability to control $n$, as (A2) must still hold with $x_A = 0$, which implies
\[
0 = \frac{2h\phi (1 - \phi)q - 2k_F}{(1 - \phi)(1 - \phi)n} - (1 + \lambda)x_F - \ell.
\]

98
After replacing \( x_F \) with (25), this becomes
\[
0 = \frac{2h[\phi + (1 - \phi)q] - 2k_F}{(1 - \phi)(1 - \bar{q})n} - \frac{2(1 + \lambda)k_F}{\bar{q}(1 - \phi)n} - \ell,
\]
which yields (27).

Finally, the government can choose to implement an irresponsible equilibrium with \( q = 0 \). In this case, any \( x_F > 0 \) creates a deadweight loss without affecting the firm’s choice of quality, so it is optimal for the government to set \( x_F \) equal to zero. Thus, we can use (A2) with \( q = 0 \) and \( x_F = 0 \) to write
\[
x_A = \frac{1}{3 + \lambda} \left( \frac{2h\phi}{(1 - \phi)n} - \ell \right).
\]
(A9)

After replacing \( x_A \) in (23) with this last expression and \( x_F \) with zero, the government’s problem becomes
\[
\max_n \frac{n}{3 + \lambda} \left[ 3h\phi - \frac{3n}{2} (1 - \phi)\ell \right].
\]
(A10)

This expression is maximized at \( n = \frac{6h\phi}{(1 - \phi)\ell} \).

To determine which equilibrium dominates for different values of \( k_F > 0 \), let us calculate the total welfare that results from each. Using (24) in (A4) yields
\[
\mathcal{W}_i(k_F) = \left( \frac{3h\bar{q}[\phi + (1 - \phi)\bar{q}] - k_F[3\bar{q} + 2\lambda(1 - \bar{q})]}{6\ell(1 - \phi)\bar{q}^2(1 - \bar{q})(3 + \lambda)} \right)^2.
\]
(A11)

We can also use \( q = \bar{q} \), \( x_F = \frac{2h\phi}{(1 - \phi)n} \), (27), and \( x_A = 0 \) in (23) to find
\[
\mathcal{W}_i(k_F) = \frac{2k_F(\bar{q}[\phi + (1 - \phi)\bar{q}] - k_F[1 + \lambda(1 - \bar{q})])}{\ell(1 - \phi)\bar{q}^2}.
\]
(A12)

We know from (A8) that the total welfare for the responsible equilibrium is \( \mathcal{W}_i(k_F) \) for \( k_F < \tilde{K}_i \), and \( \mathcal{W}_{ii}(k_F) \) for \( k_F \geq \tilde{K}_i \). Also, since \( \mathcal{W}_{ii}(k_F) \) is the solution to a constrained version (the constraint being \( x_A = 0 \)) of the problem that yields \( \mathcal{W}_i(k_F) \), we must have \( \mathcal{W}_i(k_F) \geq \mathcal{W}_{ii}(k_F) \) for all \( k_F > 0 \) (where the equality is strict everywhere except at \( k_F = \tilde{K}_i \)). The total welfare in the irresponsible equilibrium is obtained by inserting \( n = \frac{h\phi}{(1 - \phi)\ell} \) in (A10):
\[
\mathcal{W}_{iii} = \frac{3h^2\phi^2}{2\ell(1 - \phi)(3 + \lambda)}.
\]
(A13)

It is easy to verify that \( \mathcal{W}_i(0) > \mathcal{W}_{iii} \). Also,
\[
\mathcal{W}_i(h\bar{q}) = \frac{h^2\bar{q}^2(3\phi - 2\lambda)^2(1 - \bar{q})^2}{6\ell(1 - \phi)\bar{q}^2(1 - \bar{q})(3 + \lambda)} = \frac{9h^2\phi^2}{6\ell(1 - \phi)(3 + \lambda)} = \mathcal{W}_{iii}.
\]

Thus, \( \mathcal{W}_i(k_F) \) intersects \( \mathcal{W}_{iii} \) exactly once for \( k_F \in [0, h\bar{q}] \). Given this, it is straightforward to show that \( \mathcal{W}_i(k_F) \geq \mathcal{W}_{iii} \) if and only if
\[
k_F \leq \frac{3h\bar{q}[\phi(1 - \sqrt{1 - \bar{q}}) + (1 - \phi)\bar{q}]}{3\bar{q} + 2\lambda(1 - \bar{q})} \equiv \tilde{K}_i \in (0, h\bar{q}).
Similarly, we have $W_{i}(0) = 0 < W_{ii}$, and
\[
W_{i}(h\bar{q}) = \frac{2k_{F}(1 - \phi)\bar{q}}{\ell(1 - \phi)\bar{q}^{2}}
\]
\[
= -\frac{2k_{F}\ell(1 + \phi)(1 - \bar{q})}{\ell(1 - \phi)\bar{q}} < 0 = W_{ii}.
\]
Given this, it is straightforward to show that $\max_{k_{F}} W_{i}(k_{F}) > W_{ii}$ if and only if
\[
D_{ii} \equiv [\phi + (1 - \phi)\bar{q}]^{2}(3 + \lambda)^{2} - 3\phi^{2}(1 + \lambda(1 - \bar{q}))(3 + \lambda) > 0,
\]
(A14)
or equivalently,
\[
\phi < \frac{\bar{q}}{\sqrt{\frac{2}{\lambda}[(1 + \lambda(1 - \bar{q}))(1 - \bar{q})]}} \in (0, 1).
\]
(A15)

When this condition is not satisfied, the responsible equilibrium always has $x_{A} > 0$, and the proposition obtains with $\hat{k}_{i} = \hat{K}_{i}$ and $\hat{k}_{i} = \hat{K}_{i}$. When (A15) is satisfied, $W_{i}(k_{F}) \geq W_{ii}$ if and only if
\[
\frac{h\bar{q}([\phi + (1 - \phi)\bar{q}]/[(3 + \lambda)^{2} - 3\phi^{2}(1 + \lambda(1 - \bar{q}))(3 + \lambda) + \sqrt{D_{ii}}])}{2(3 + \lambda)[1 + \lambda(1 - \bar{q})]} \leq k_{F} \leq \frac{h\bar{q}([\phi + (1 - \phi)\bar{q}]/[(3 + \lambda)^{2} - 3\phi^{2}(1 + \lambda(1 - \bar{q}))(3 + \lambda) - \sqrt{D_{ii}}])}{2(3 + \lambda)[1 + \lambda(1 - \bar{q})]}
\]
\[
\equiv \hat{k}_{i} \in (0, h\bar{q}).
\]
Then, we can also show that $\hat{k}_{i} < \hat{K}_{i}$ if and only if
\[
\phi > \frac{2(3 + \lambda)\bar{q}\sqrt{1 - \bar{q}}}{3(2 - \bar{q}) - 6(1 - \bar{q})\sqrt{1 - \bar{q}} + 2\lambda(1 - \bar{q})(2 - \sqrt{1 - \bar{q}})} \in (0, 1).
\]

When this last condition holds, since $W_{i}(k_{F}) \geq W_{i}(k_{F})$ for all $k_{F} > 0$ and since we can verify that $W_{i}(k_{F}) < 0$ for all $k_{F} \geq \hat{K}_{i}$, the responsible equilibrium always has $x_{A} > 0$, and the proposition obtains with $\hat{k}_{i} = \hat{K}_{i}$ and $\hat{k}_{i} = \hat{K}_{i}$. Otherwise, the responsible equilibrium has $x_{A} = 0$ for $k_{F} \in \hat{K}_{i}$, $\hat{k}_{i}$, and so the equilibrium obtains with $\hat{k}_{i} = \hat{K}_{i}$ and $\hat{k}_{i} = \hat{K}_{i}$.

**Proof of Proposition 3**

(i) It is clear from (24) that $n$ is increasing in $\phi$ and $h$, and decreasing in $k_{F}$, $\ell$, and $\lambda$. To establish that $n$ is increasing in $\bar{q}$, let us rewrite (24) as
\[
\frac{3h(\phi + (1 - \phi)\bar{q}) - k_{F}[3 - 2\lambda + \frac{\phi}{\bar{q}}]}{3(1 - \phi)(1 - \bar{q})}.
\]
The numerator (denominator) of this expression is clearly increasing (decreasing) in $\bar{q}$, establishing the result. The comparative statics for $a = 1 - n$ trivially follow. Also, we know from (28) and the surrounding discussion that first-best is obtained when $\lambda = 0$. The fact that the number $n$ of sales (the advice level $a$) is smaller (larger) than in first-best therefore follows from the comparative statics in $\lambda$.

(ii) After adding $x_{F}$ to both sides of (26), we get
\[
x_{A} + x_{F} = \frac{1}{3 + \lambda} \left( \frac{2h(\phi + (1 - \phi)\bar{q}) - 2k_{F}[1 - \phi(1 - \bar{q})]}{(1 - \phi)(1 - \bar{q})n} + 2x_{F} - \ell \right).
\]
We can then replace $x_{F}$ with (25). After simplifying, we have
\[
x_{A} + x_{F} = \frac{1}{3 + \lambda} \left( \frac{2h(\phi + (1 - \phi)\bar{q}) + k_{F}(4 - 6\bar{q})}{(1 - \phi)(1 - \bar{q})n} - \ell \right).
\]
Finally, we can replace \( n \) with (24) and simplify to get
\[
x_{F} = \frac{\ell}{3 + \lambda} \left( \frac{3h}[\phi + (1 - \phi)\overline{q}] + k_{F}[12 - 15\overline{q} + 2\lambda(1 - \overline{q})] + k_{F}[3\overline{q} + 2\lambda(1 - \overline{q})] \right) \tag{A16}
\]
It is clear from this expression that \( x_{A} + x_{F} \) is decreasing in \( \overline{q}, \phi, \) and \( h, \) and increasing in \( k_{F} \) and \( \ell. \)

(iii) We can readily see from (A7) that \( x_{A} \) is increasing in \( \overline{q}, \phi, \) and \( h, \) decreasing in \( k_{F}, \) and proportional to \( \ell. \) After we insert (24) in (25) and simplify, we have
\[
x_{F} = \frac{6\ell(1 - \overline{q})k_{F}}{3h}[\phi + (1 - \phi)\overline{q}] - k_{F}[3\overline{q} + 2\lambda(1 - \overline{q})].
\]
This quantity is clearly decreasing in \( \overline{q}, \phi, \) and \( h, \) increasing in \( k_{F}, \) and proportional to \( \ell. \) Thus, the ratio \( \frac{x_{F}}{x_{A}} \) (as well as the ratio \( \frac{x_{A}}{x_{A} + x_{F}}, \) by implication) is increasing in \( \overline{q}, \phi, \) and \( h, \) decreasing in \( k_{F}, \) and unaffected by changes in \( \ell. \) □

**Proof of Proposition 4**

This proof proceeds along the same lines as the proof of Proposition 2. The firm must choose \( x_{F} \geq 0 \) and \( x_{A} \geq 0 \) in order to maximize (29), subject to the firm’s equilibrium choices of \( q \) and \( w \) (in Sections 3.2 and 3.3) and the agent’s equilibrium choices of \( n \) and \( a \) (in Section 3.1). After we insert \( w \) from (13) and \( p \) from (20) in (29), and replace \( 1 - a \) with \( n, \) the firm’s problem becomes
\[
\max_{x_{A}, x_{F}} n\left( [\phi + (1 - \phi)\overline{q}] - \frac{n}{2}(1 - \phi)(1 - \overline{q})[\ell - (1 - \lambda)(x_{A} + x_{F})] - n(1 - \phi)(1 - \overline{q})x_{A} - \frac{n}{2}(1 - \phi)(1 - \overline{q})x_{F} - k_{F}I_{(q = \overline{q})} \right) - \frac{n}{2}(1 - \phi)(1 - \overline{q})[\ell + (1 + \lambda)x_{A} + \lambda x_{F}] - k_{F}I_{(q = \overline{q})}. \tag{A17}
\]
Let us first conjecture a responsible equilibrium in which the firm chooses \( q = \overline{q}. \) We can use (22) with \( q = \overline{q} \) to write \( x_{A} \) as a function of \( n \) and \( x_{F}: \)
\[
x_{A} = \frac{1}{3 + \lambda} \left( \frac{2h}[\phi + (1 - \phi)\overline{q}] - 2k_{F}(1 - \phi)(1 - \overline{q})n \right) - (1 + \lambda)x_{F} - \ell \tag{A18}
\]
This is (32). As in the proof of Proposition 2, \( x_{F} \) must satisfy (16) with equality and so is given by (31). After we replace \( x_{A} \) and \( x_{F} \) in (A17) with their expressions in (32) and (31), the firm’s problem simplifies to
\[
\max_{n} n\left( \frac{n}{3 + \lambda} \left( 2h\phi + (1 - \phi)\overline{q} - n(1 - \phi)(1 - \overline{q})[\ell + k_{F}\overline{q}(1 - 3\overline{q} - \lambda(1 - \overline{q}))] \right) \right). \tag{A19}
\]
The first-order condition with respect to \( n \) yields (30). This \( n \) is positive since
\[
2h\overline{q}[\phi + (1 - \phi)\overline{q}] + k_{F}[1 - 3\overline{q} - \lambda(1 - \overline{q})] > 2h\overline{q}[\phi + (1 - \phi)\overline{q}] - 2k_{F}\overline{q} > 0,
\]
where the first inequality follows from \( \lambda < 1 \) and the last inequality follows from (a2). To show that (30) is smaller than \( 1 - \tilde{a}_{q} \) [with \( \tilde{a}_{q} \) defined in (7)], notice that this is equivalent to showing that
\[
2h\overline{q}[\phi + (1 - \phi)\overline{q}] + k_{F}[1 - \lambda - \overline{q}(3 - \lambda)] < 4h\overline{q}[\phi + (1 - \phi)\overline{q}] - 4k_{F}\overline{q},
\]
which in turn is equivalent to
\[
-2h\overline{q}[\phi + (1 - \phi)\overline{q}] + k_{F}[1 - \lambda + \overline{q} + \overline{q}\overline{q}] < 0.
\]
This inequality holds, as \( (a2) \) implies

\[
-2h\bar{q}(\phi + (1 - \phi)\bar{q}) + k_F[1 - \lambda + \bar{q} + \lambda\bar{q}]
\]

\[
< -2h\bar{q}(\phi + (1 - \phi)\bar{q}) + h\bar{q}[1 - \lambda + \bar{q} + \lambda\bar{q}]
\]

\[
= -h\bar{q}[(2\phi - 1)(1 - \bar{q}) + \lambda(1 - \bar{q})]
\]

\[
\leq -h\bar{q}\lambda(1 - \bar{q}) < 0,
\]

where the next-to-last inequality results from \( \phi \geq \frac{1}{2} \). Thus, we have \( n < 1 - \bar{q} \bar{q} \). Given that \( n > 0 \), it is clearly the case that \( (31) \) is positive. We next verify that \( (32) \) is also positive for \( k_F \) sufficiently small. By inserting \( (31) \) in \( (32) \) and simplifying, we get \( (A6) \). Now, let us insert \( (30) \) for \( n \) in \( (A6) \):

\[
x_A = \frac{\ell}{3 + \lambda} \left( \frac{4h\bar{q}(\phi + (1 - \phi)\bar{q}) - 4k_F[1 + \lambda(1 - \bar{q})]}{n} - 1 \right)
\]

\[
= \frac{\ell}{3 + \lambda} \left( \frac{2h\bar{q}(\phi + (1 - \phi)\bar{q}) - k_F[5 - 3\bar{q} + 3\lambda(1 - \bar{q})]}{2h\bar{q}[\phi + (1 - \phi)\bar{q}] + k_F[1 - \lambda - \bar{q}(3 - \lambda)]} \right)
\]

\[
= \frac{\ell}{3 + \lambda} \left( 1 - \frac{2k_F(3 + \lambda)(1 - \bar{q})}{2h\bar{q}(\phi + (1 - \phi)\bar{q}) + k_F[1 - \lambda - \bar{q}(3 - \lambda)]} \right).
\]

This last quantity is positive if and only if

\[
2h\bar{q}(\phi + (1 - \phi)\bar{q}) + k_F[1 - \lambda - \bar{q}(3 - \lambda)] > 2k_F(3 + \lambda)(1 - \bar{q}),
\]

which is equivalent to

\[
2h\bar{q}(\phi + (1 - \phi)\bar{q}) > k_F[2(3 + \lambda)(1 - \bar{q}) + 3\bar{q} + 2\lambda(1 - \bar{q})],
\]

and in turn to

\[
k_F < \frac{2h\bar{q}(\phi + (1 - \phi)\bar{q})}{2 + 3(1 + \lambda)(1 - \bar{q})} \equiv \bar{K}_0 \in (0, h\bar{q}).
\]

The firm can also implement the responsible equilibrium for larger values of \( k_F \) by setting \( x_A = 0 \) and \( x_F \) sufficiently large for the firm to commit to choosing \( q = \bar{q} \), i.e., \( x_F \), as in \( (31) \). In this case, the firm loses its ability to control \( n \), as \( (A2) \) must still hold with \( x_A = 0 \) which, as in the proof of Proposition 2, implies \( (33) \).

Finally, the firm can choose to implement an irresponsible equilibrium with \( q = 0 \). As in the proof of Proposition 2, it is optimal for the firm to set \( x_F \) equal to zero, and we can use \( (A2) \) with \( q = 0 \) and \( x_F = 0 \) to write \( x_A \) as in \( (A9) \). After replacing \( x_A \) and \( x_F \) in \( (A17) \) with \( (A9) \) and zero, respectively, the firm’s problem becomes

\[
\max \frac{n}{3 + \lambda} \left[ 2h\phi - n(1 - \phi)\ell \right].
\]

This expression is maximized at \( n = \frac{h\phi}{\ell(1 - \phi)} \).

To determine which equilibrium dominates for different values of \( k_F > 0 \), let us calculate the firm value that results from each. Using \( (30) \) in \( (A19) \) yields

\[
\mathcal{V}_c(k_F) = \frac{(2h\bar{q}(\phi + (1 - \phi)\bar{q}) + k_F[1 - \lambda - \bar{q}(3 - \lambda)])^2}{4(1 - \phi)\bar{q}^2(1 - \bar{q})(3 + \lambda)}.
\]

We can also use \( q = \bar{q} \), \( x_F = \frac{2k_F}{\phi(1 - \phi)\ell} \) \( (33) \), and \( x_A = 0 \) in \( (A17) \) to find

\[
\mathcal{V}_u(k_F) = \frac{k_F(h\bar{q}(\phi + (1 - \phi)\bar{q}) - k_F[1 + \lambda(1 - \bar{q})])}{\ell(1 - \phi)\bar{q}^2}.
\]
We know from (A21) that the firm value for the responsible equilibrium is \( V_i(k_F) \) for \( k_F < \bar{K}_i \), and \( V_i(k_F) \) for \( k_F \geq \bar{K}_i \). Also, since \( V_i(k_F) \) is the solution to a constrained version (the constraint being \( x_A = 0 \)) of the problem that yields \( V_i(k_F) \), we must have \( V_i(k_F) \geq V_i(k_F) \) for all \( k_F > 0 \) (where the equality is strict everywhere except at \( k_F = \bar{K}_i \)). The firm value in the irresponsible equilibrium is obtained by inserting \( n = \frac{h q}{(1 - \phi) q} \) in (A22):

\[
V_{\text{ir}} = \frac{h^2 \phi^2}{\ell (1 - \phi)(3 + \lambda)}.
\]  

(A25)

It is easy to verify that \( V_i(0) > V_{\text{ir}} \), and that \( V_i(k_F) \) is strictly increasing for all \( k_F > 0 \) if \( q < \frac{1}{1 - \phi} \). If instead \( q \geq \frac{1}{1 - \phi} \), it is readily verified that \( V_i(0) < 0 \) and

\[
V_i(hq) = [1 - \lambda - \bar{q}(3 - \lambda)]\frac{h \bar{q}(2[\phi + (1 - \phi) \bar{q}] + 1 - \bar{q}(3 - \lambda))}{2(1 - \phi)\bar{q}^2(1 - \bar{q})(3 + \lambda)}
\]

\[\quad = - h(3 - \lambda)\left(\bar{q} - \frac{1 - \lambda}{3 - \lambda}\right) \frac{2\phi + (1 - \lambda)}{2(1 - \phi)\bar{q}^2(3 + \lambda)} < 0.
\]

Thus, \( V_i(k_F) \) intersects \( V_{\text{ir}} \) at most once for \( k_F \in \{0, h\bar{q}\} \), and it is straightforward to verify that \( V_i(k_F) \geq V_{\text{ir}} \) if and only if

\[
k_F \leq \min\left\{hq, \frac{2h\bar{q}(\phi(1 - \sqrt{1 - \bar{q}}) + (1 - \phi)\bar{q})}{\lambda + \bar{q}(3 - \lambda) - 1}\right\} = \bar{K}_i.
\]

Similarly, we have \( V_i(0) = 0 < V_{\text{ir}} \), and

\[
V_i(hq) = \frac{2k_F h\bar{q}(\phi + (1 - \phi)\bar{q}) - [1 + \lambda(1 - \bar{q})]}{\ell(1 - \phi)\bar{q}^2}
\]

\[\quad = - \frac{2k_F h(1 + \lambda - \phi)(1 - \bar{q})}{\ell(1 - \phi)\bar{q}^2} < 0 = V_{\text{ir}}.
\]

Given this, it is straightforward to show that

\[
\Delta_{\text{ir}} = [\phi + (1 - \phi)\bar{q}]^2(3 + \lambda)^2 - 2\phi^2[1 + \lambda(1 - \bar{q})](3 + \lambda) > 0,
\]

and that \( V_i(k_F) \geq V_{\text{ir}} \) if and only if

\[
\frac{h \bar{q}[(\phi + (1 - \phi)\bar{q})(3 + \lambda) - \sqrt{\Delta_{\text{ir}}})}{2(3 + \lambda)(1 + \lambda(1 - \bar{q}))} \leq k_F \leq \frac{h \bar{q}[(\phi + (1 - \phi)\bar{q})(3 + \lambda) + \sqrt{\Delta_{\text{ir}}})}{2(3 + \lambda)(1 + \lambda(1 - \bar{q}))} = \bar{K}_i (0, h\bar{q}).
\]

If \( \bar{q} < \frac{1}{1 - \phi} \), then we have \( \bar{K}_i = h\bar{q} > \bar{K}_i \). Since \( V_i(k_F) \geq V_i(k_F) \) for all \( k_F > 0 \), the responsible equilibrium has \( x_A = 0 \) for \( k_F \in [\bar{K}_i, \bar{K}_i] \), and so the result obtains with \( \bar{K}_i = \bar{K}_i \) and \( \bar{K}_i = \bar{K}_i \). Suppose instead that \( \bar{q} \geq \frac{1}{1 - \phi} \). Then, we can show that \( \bar{K}_i > \bar{K}_i \) if and only if

\[
\phi < \frac{2(3 + \lambda)\bar{q}\sqrt{1 - \bar{q}}}{2 + 3(1 - \bar{q}) - 6(1 - \bar{q})\sqrt{1 - \bar{q}} + \lambda(1 - \bar{q})(3 - 2\sqrt{1 - \bar{q}})},
\]

and, when this condition holds, we again have the result with \( \bar{K}_i = \bar{K}_i \) and \( \bar{K}_i = \bar{K}_i \). Otherwise, the responsible equilibrium always has \( x_A > 0 \), and the proposition obtains with \( \bar{K}_i = \bar{K}_i \) and \( \bar{K}_i = \bar{K}_i \). ■
Proof of Proposition 5
Let us denote the equilibrium quantities that result from the government (firm) choosing the penalties with a superscript “G” (“F”), and the first-best quantities with a star superscript.

Let us first prove part (ii) of this proposition. Using (9) and (30), it is the case that
\[ n^F > n^* \] if and only if
\[ \frac{h[\phi + (1 - \phi)q] - k_F\bar{q}}{\ell(1-\phi)(1-\bar{q})} > \frac{2h[\phi + (1 - \phi)\bar{q}] + k_F[1 - 3\bar{q} - \lambda(1 - \bar{q})]}{2\ell(1-\phi)\bar{q}(1-\bar{q})}. \]

After simplification, this inequality reduces to
\[ 3 + \lambda(1 - \bar{q}) > 0, \]
which is clearly true. Also,
\[ \lim_{\lambda \to 0} n^F = \frac{2h[\phi + (1 - \phi)\bar{q}] + k_F[1 - 3\bar{q}]}{2\ell(1-\phi)\bar{q}(1-\bar{q})} = \frac{2h[\phi + (1 - \phi)\bar{q}] - 2k_F\bar{q} + k_F(1 - \bar{q})}{2\ell(1-\phi)\bar{q}(1-\bar{q})} = n^* + \frac{k_F(1 - \bar{q})}{2\ell(1-\phi)\bar{q}(1-\bar{q})} > n^*. \]

Since \( a = 1 - n \), these results imply that \( a^F < a^* \) and \( \lim_{\lambda \to 0} a^F \) is strictly smaller than \( a^* \), establishing part (ii) of the proposition.

We know from part (i) of Proposition 3 that \( n^G < n^* \). Thus, the results from part (ii) proved above imply that \( n^F > n^G \) and \( a^F < a^G \). Because \( x_F = \frac{2h\phi + (1 - \phi)\bar{q}}{\ell/\bar{q}} \) [see (25) and (31)], this implies that \( x_F^G < x_F^G \). To show that \( x_A^G < x_A^G \), let us use the expressions for these quantities in (A7) and (A20) to calculate
\[ x_A^G - x_A^F = \frac{\ell}{3 + \lambda} \left( \frac{2k_F(3 + \lambda)(1 - \bar{q})}{2h[\phi + (1 - \phi)\bar{q}] + k_F[1 - \lambda - \bar{q}(3 - \lambda)] - \frac{2k_F(3 + \lambda)(1 - \bar{q})}{3h[\phi + (1 - \phi)\bar{q}] - k_F[3\bar{q} + 2\lambda(1 - \bar{q})]} \right) \]
\[ = \frac{2(k_F(1 - \bar{q})[h[\phi + (1 - \phi)\bar{q}] + k_F[1 - \lambda - \bar{q}(3 - \lambda)] \times (3h[\phi + (1 - \phi)\bar{q}] - k_F[3\bar{q} + 2\lambda(1 - \bar{q})]))}{(2h[\phi + (1 - \phi)\bar{q}] + k_F[1 - \lambda - \bar{q}(3 - \lambda)]) \times (3h[\phi + (1 - \phi)\bar{q}] - k_F[3\bar{q} + 2\lambda(1 - \bar{q})])}. \]

Therefore, to show the desired result, we need to show that the expression in curly brackets in the numerator of this last expression,
\[ h[\phi + (1 - \phi)\bar{q}] - k_F[1 + \lambda(1 - \bar{q})], \]
(A27)
is positive for all \( k_F \in [0, \bar{c}_A] \). Recall from the proof of Proposition 4 that \( \bar{c}_A = \min(\bar{K}_A, \bar{K}_f) \).
Thus, since (A27) is decreasing in \( k_F \), it is sufficient to verify that it is greater than zero when evaluated at \( k_F = \bar{K}_A \). This is indeed the case, as
\[ h[\phi + (1 - \phi)\bar{q}] - k_A[1 + \lambda(1 - \bar{q})] \]
\[ = h[\phi + (1 - \phi)\bar{q}] - \frac{2h[\phi + (1 - \phi)\bar{q}]}{2 + 3(1 + \lambda)(1 - \bar{q})}[1 + \lambda(1 - \bar{q})] \]
\[ = h[\phi + (1 - \phi)\bar{q}] \left( 1 - \frac{2[1 + \lambda(1 - \bar{q})]}{2 + 3(1 + \lambda)(1 - \bar{q})} \right) \]
\[ = h[\phi + (1 - \phi)\bar{q}] \left( \frac{(3 + \lambda)(1 - \bar{q})}{2 + 3(1 + \lambda)(1 - \bar{q})} \right) > 0. \]

The result that \( p^F < p^G \) is implied by \( a^F < a^G \), \( x_A^F < x_A^G \), \( x_F^F < x_F^G \), and the fact that \( p \) must satisfy (20) [with \( q = \bar{q} \)], which is increasing in \( a, x_A, \) and \( x_F \). Finally, to show that \( w^F < w^G \), notice from (13) that \( w \) must equal \( (1 - \phi)(1 - \bar{q})n_A x_A \), whether it is set by the government or the firm. Thus, it is sufficient to show that \( n^F x_A^F < n^G x_A^G \). From the proofs of
Propositions 2 and 4, we know that $x_A$ must satisfy (A6). After multiplying this equation by $n$, we have

$$nx_A = \frac{1}{3 + \lambda} \left( \frac{2\phi q + (1 - \phi)\bar{q} - 2k_F[1 + \lambda(1 - \bar{q})]}{(1 - \phi)(1 - \bar{q})} - nF \right),$$

which is decreasing in $n$. Thus, the result follows from $n^F > n^G$. ■

References


