Strategic price complexity in retail financial markets

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Abstract

There is mounting empirical evidence to suggest that the law of one price is violated in retail financial markets: there is significant price dispersion even when products are homogeneous. Also, despite the large number of firms in the market, prices remain above marginal cost and may even rise as more firms enter. In a non-cooperative oligopoly pricing model, I show that these anomalies arise when firms add complexity to their price structures. Complexity increases the market power of the firms because it prevents some consumers from becoming knowledgeable about prices in the market. In the model, as competition increases, firms tend to add more complexity to their prices as a best response, rather than make their disclosures more transparent. Because this may substantially decrease consumer surplus in these markets, such practices have important welfare implications.

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1. Introduction

Price formation in retail financial markets deviates from the predictions of standard price theory in several important ways. The law of one price is violated: significant price dispersion is present when goods and services are homogeneous. Despite the large number of firms in each market, prices do not converge to marginal cost. Even when new firms enter the industry, prices often do not decrease and may in fact rise. These pricing irregularities have been documented empirically in the markets for S&P Index funds (Hortacsu and Syverson, 2004), money market funds (Christoffersen and Musto, 2002), mutual funds (Bergstresser, Chalmers, and Tufano, 2007), retail municipal bonds (Green, Hollifield, and Schuhruff, 2007; Green, 2007), credit cards (Ausubel, 1991), conventional fixed-rate mortgages (Baye and Morgan, 2001), life annuities (Mitchell, Poterba, Warsawsky, and Brown, 1999), and term life insurance (Brown and Goolsbee, 2002).

What is responsible for this seeming departure from classic microeconomics? The answer that I explore builds on the fifty year-old observation by Scitovsky (1950) that ignorance is a source of oligopoly power. Producers of retail financial products create ignorance by making their prices more complex, thereby gaining market power and the ability to increase industry profits. Clearly, many of the households who purchase retail financial products do not understand what they are buying and how much they are paying for these goods (e.g., Capon, Fitzsimons, and Prince, 1996; Alexander, Jones, and Nigro, 1998; Barber, Odean, and Zheng, 2005; Agnew and Szykman, 2005) and access to financial advice does not appear to rectify this problem (Bergstresser, Chalmers, and Tufano, 2007). Importantly,
however, there appears to be a significant gap between investor knowledge about the financial instruments themselves and their understanding of industry fees. For example, in the NASD Investor Literacy Survey (2003), 84% of market participants understood the relative riskiness of various bonds, but only 21% knew what a “no-load mutual fund” is. In fact, approximately one-third of the participants surveyed believed that the term no-load implies that there are no fees charged whatsoever.

In this paper, I consider the following important questions: How does complexity affect price formation in the market? How do firms optimally add complexity to their price structures to maximize profits? How do these optimal pricing policies change as industry competition increases? What is the potential effect of these policies on consumer knowledge and prices in the market?

Financial institutions may add complexity to their prices in several ways. First, they can make it more difficult for households to become informed by partitioning prices into direct fees and indirect involuntary surcharges. This practice makes understanding prices more challenging as it places the responsibility on the consumer to appreciate all of the key price components and compute the actual price of the product. Second, complexity may be added when firms devise new technical language for their price disclosures. If firms in the industry use different methods of disclosure, this makes it more difficult for consumers to compare prices.

Complexity may also involve leaving out important information in a disclosure. This aspect makes it tougher for low-price firms to credibly signal their advantage because advertising is a mechanism for signal jamming. For example, suppose a low-price mutual fund makes a statement that they have low management fees and no loads. A higher-priced fund can advertise that they have no management fees and no loads, even though they charge high 12b-1 fees and other indirect costs. Used in this way, complexity makes it harder for consumers to identify the best deals in the market.

In the paper, I analyze a two-stage pricing complexity game in which homogeneous firms produce an identical financial product and compete on price for market share. In the first period, firms simultaneously choose their prices (mutual fund fees, interest rates, etc.) and the complexity of their price structures. The complexity that one particular firm adds may increase the difficulty in evaluating their own price disclosure and comparing prices in the market, but does not affect the ability for consumers to evaluate the disclosures of competing firms. Based on the complexity choices of the firms, a fraction of consumers become informed about prices (experts), whereas the remainder remain uninformed. In the second period, the experts purchase the good from the low-priced firm, whereas uninformed consumers choose randomly from all of the firms.

In equilibrium, price dispersion arises because the firms compete strategically for market share from both types of consumers. This feature is also present in other models of search (e.g. Varian, 1980; Stahl, 1989), but arises here based on each firm’s complexity decision (to be discussed shortly). The firm with the lowest price captures the entire share of expert consumers. All of the firms, however, receive some demand from the uninformed. The firms never charge marginal cost because they gain positive expected profits from sales to uninformed consumers. Also, it is impossible to have a one-price equilibrium in which all firms charge the same prices for their products. If they did so, one firm could undercut their competitors by a small amount and gain the entire market share from the expert consumers. So, equilibrium prices are strictly higher than marginal cost and there is always a non-degenerate distribution of prices (price dispersion).

Price complexity in the industry is determined through strategic interaction between the firms. In equilibrium, all firms enjoy a positive rent from having some degree of price complexity in the industry and preventing some consumers from becoming informed. However, low-price firms desire less complexity than high-price firms. Since the low-price firms want consumers to know that they have the cheapest prices, they want pricing in the industry to be reasonably clear. Adding clarity allows them to undercut their rivals and gain market share. They do not want pricing in the industry to be too clear, however, as total clarity would erode industry rents altogether. In contrast, high-price firms desire more complexity. As pricing in the industry becomes more difficult to appreciate, the fraction of uninformed consumers rises, thereby increasing the market share that high-price firms receive. Decreasing industry price transparency is the way high-price firms gain market share.

After deriving the equilibrium of the game, I consider how increased competition affects the way in which complexity evolves in the market. I find that increased competition makes it more likely that firms make their price disclosures opaque. The intuition is as follows. When more firms compete for market share, the probability that they receive demand from the expert consumers decreases. As a best-response (i.e., to maximize expected profits), firms tend to increase complexity in order to maximize the revenues they receive when they do not have the lowest price (when they lose the share of experts). Therefore, as competition rises, attempting to increase the fraction of uninformed consumers improves their expected profitability. The fact that more firms tend to add complexity when industry concentration decreases may induce a drop in the fraction of informed consumers, which in turn may increase producer surplus in the market. That is, unless there are other mechanisms present that make it easier for consumers to become knowledgeable as the industry grows (e.g., consumer organizations or government-sponsored education), then industry rents may rise as the market becomes more competitive. In this light, it is not surprising that Hortacsu and Syverson (2004) show that entry into the S&P index fund industry in 1995–1999 was associated with a rightward shift in the distribution of prices.

The analysis in this paper yields several novel empirical implications. For example, since the model implies that complexity is an important source of value for firms, changes in complexity should be positively correlated with firm profitability, ceteris paribus. Also, the
model predicts that as competitive pressures rise in an industry (more producers enter the market), firms will respond by adding more complexity to their prices. This implies a negative correlation between industry concentration and price complexity. Though I do not test these predictions in this paper, they may be tested either cross-sectionally or with a time-series, perhaps using content analysis (e.g., Holsti, 1969; Tetlock, 2007) to quantify the amount of complexity present in various price disclosures.

This paper is of general economic interest as price dispersion has also been documented empirically in the markets for prescription drugs (Sorensen, 2000), books (Clay, Krishnan, and Wolff, 2001; Chevalier and Goolsbee, 2003), and computer memory modules (Ellison and Ellison, 2005). The paper also adds to an extensive literature on oligopoly pricing with consumer search (e.g., Diamond, 1971; Salop and Stiglitz, 1977; Varian, 1980; Stahl, 1989). In many existing models, the fraction of consumers who conduct complete search is exogenously given and the firms are unable to affect the search environment except through the prices they choose. Indeed, price dispersion arises in many of the models, but its source and its severity are determined by the exogenous parameters imposed in each model. In contrast, I provide a model in which firms endogenously affect the proportion of consumer types by altering the search environment. The complexity that firms add affects the proportion of consumers who become knowledgeable, and in turn affects what they will pay for goods in the market. The “hide and seek” pricing model that I develop is new to this literature, and provides insights into the role of information in achieving industry price dispersion.

There is a growing related literature that evaluates how rational firms strategically set prices in response to the shortcomings of their consumer population. Heidhues and Koszegi (2005) study a monopolist’s optimal pricing behavior when their consumers are loss averse. Perloff and Salop (1985) derive optimal pricing strategies given that all consumers make errors in calculating their value for the good. Gabaix, Laibson, and Li (2005) use extreme value theory to generalize the Perloff-Salop derivations. Gabaix and Laibson (2006) derive a “shrouded attributes equilibrium” in which prices are set and voluntary add-ons are chosen, all based on a given fraction of the consumers who are myopic. The common theme in all of these papers is that the proportion of the consumers who are biased or less informed is exogenously given. In contrast, in this paper, this proportion evolves endogenously as the actions of the firms directly affect the proportion of experts who are present in the market.

The rest of the paper is organized as follows. In Section 2, I introduce the two-stage pricing complexity game. In Section 3, I characterize the strategic behavior of the firms and prove existence of a symmetric mixed-strategy Nash equilibrium for the game. In Section 4, I characterize the effect that entry has on complexity. Section 5 concludes. The Appendix contains all of the proofs.

2. The market for retail financial products

Consider a market in which \( n \) firms, indexed by \( j \in \{1, \ldots, n\} \), produce a homogeneous retail financial product. The product may be used by households to finance the purchase of consumption goods (for example, a credit card) or as an investment vehicle to maximize lifetime utility (for example, an index fund). The firms face zero marginal costs and have no capacity constraints. The only potential difference between the firms is the price that they charge and the complexity that they add to their price structures. Restricting the firms to produce an undifferentiated good is not just for technical convenience. Rather, if complexity in fee structures causes failure of competition and price dispersion in the market, adding heterogeneity in the product attributes will only make price dispersion more likely.

In the market, there is a unit mass of consumers \( M \) who each have unit demand for the retail good. Every consumer \( i \) is risk-neutral and maximizes the expected payoff from their purchase. Their utility is given by

\[
U_i = v - p_i,
\]

where \( v \) is the fundamental value of the product and \( p_i \) is the price that they pay. The value \( v \) is commonly known among the consumers and can be considered the monopoly price for the good. Because consumers receive \( v \) when they purchase the good from any firm, maximizing utility in this market is equivalent to minimizing the price that they pay.

Based on the actions of the firms, consumers are divided into two groups: financial experts (fraction \( \mu \)) and uninformed consumers (fraction \( 1 - \mu \)). That is, how complex the firms make their disclosures (to be specified shortly) endogenously affects how educated the consumer population is about their purchases. Experts are those consumers who become fully informed about the prices in the market and purchase the good at the lowest price available, \( p_{\text{min}} = \min(p_j)_{j=1}^n \). In contrast, uninformed consumers are those who remain uneducated about prices and purchase the good from a randomly chosen firm. As such, the probability that an uninformed consumer purchases the good from any one firm is \( 1/n \) and the expected price they pay is \( \bar{p} = (1/n)\sum_{j=1}^n p_j \). So that all uninformed consumers are rationally willing to participate in the market, I simplify the analysis and restrict the firms to choose prices \( p_j \in [0, v] \).

The pricing complexity game is, therefore, a two-stage game (Fig. 1), which proceeds as follows. In the first period \( t = 1 \), the firms simultaneously set prices for the product and decide how complicated to make their price structure. Each firm chooses \( p_j \in [0, v] \) and \( k_j \in [k, \bar{k}] \) where \( p_j \) is the actual (total) price that firm \( j \) charges and \( k_j \) is a measure of how difficult it is to sift through the price on all dimensions. I assume that the firms may choose any value for \( k_j \) in \([k, \bar{k}] \) without paying a cost for doing so. The goal is to generate strategic choices for \( k_j \) that are independent of an exogenously imposed cost function. I define \( \Sigma_j = [0, v] \times [k, \bar{k}] \) to be the strategy space for firm \( j \) and \( \sigma_j \in \Sigma_j \) to be firm \( j \)'s (mixed) strategy over prices and
complexity. In any Nash equilibrium, the strategies of the firms are given by the vector $\sigma^* = (\sigma^1, \ldots, \sigma^m)$. The effect of complexity on the consumer population is captured mathematically as follows. The proportion of experts $\mu$ is determined by the multivariate map

$$\mu : [k, R]^n \rightarrow (0, 1)$$

such that $\mu(k_1, \ldots, k_n) \in C^2$, $\partial \mu / \partial k_j < 0$ for all $j$, and $\nabla^2 \mu / \partial k_j \partial k_l = 0$ for all $j, l \neq j \in N$. The lower bound on $\mu$, when all firms maximize their complexity and choose $k$, is denoted by $\mu_{\text{min}}$. Likewise, the upper bound on $\mu$ is denoted by $\mu_{\text{max}}$. The set of restrictions that $\partial \mu / \partial k_j < 0$ for all $j$ implies that as any one firm makes its price more difficult to evaluate, it makes the whole industry harder to analyze, and thereby lowers the fraction of experts. The last set of restrictions ($\nabla^2 \mu / \partial k_j \partial k_l = 0$), however, implies that the complexity of one firm’s price does not affect the inherent difficulty in evaluating a competing firm’s offer.

The map in Eq. (1) captures the idea that complexity choices by individual firms not only make it difficult to understand the price that is quoted by that particular firm, but also may make it more difficult to compare prices among firms. For example, suppose that one firm discloses their fees and uses particular technical language that a consumer must learn to interpret. A competing firm (say, Firm 2) may adopt the same language, or they may choose (or devise) an alternative form of disclosure, which the consumer must also learn. If Firm 2 devises an alternative form of disclosure, this will not change the ability of a consumer to understand Firm 1’s price, but it does make it harder for the consumer to become fully knowledgeable and compare prices. Therefore, while an individual firm’s complexity choice may add difficulty to the overall task of becoming informed, it does not magnify the effect of other firms’ complexity choices on the cost to become informed.

At $t = 2$, consumers make purchases based on their type. The firm (or firms) with the low price receives their share of the demand from the entire mass of experts as well as a $(1/n)$ share of the demand from uninformed consumers. The other firms receive only $1/n$ of the uninformed consumers’ demand.

Partitioning consumers into two groups based on their knowledge of prices is standard in the search literature (e.g., Salop and Stiglitz, 1977; Varian, 1980) and has been referred to as an “all or nothing” search process or a “clearinghouse” search model (Baye, Morgan, and Scholten, 2006). It is typical in this literature to consider either that the fraction of informed buyers is given exogenously by a constant $\mu$, or evolves solely based on prices. This is where the analysis in this paper departs from standard approaches, in that I assume that the firms may influence how informed the consumer population is by affecting the quality of information that they are given. As such, the reduced-form map in (1) is meant to capture the fact that individuals are adversely affected by complexity and that the education level (about financial products) in the population drops when prices become less transparent.

An inherent externality that arises in all or nothing search is that every firm’s complexity choice affects the cost of the entire analysis (and therefore the fraction of experts) because it is mandatory to compare every firm’s price to all of the others. This externality also arises in other types of search models (e.g., sequential search), however, and the results that follow are not unique to an all or nothing search process. It is also important to note that the model could be generalized to include partially-informed consumers. For technical simplicity, I do not include a fraction of consumers who narrow the field of choices (e.g., use adaptive decision-making procedures like rules of thumb or heuristics). As will become clear in Section 3, only two consumer segments are required to generate price dispersion and the competitive effects described in the paper. Adding a third group would segment the market further and make the model more complicated, but would not qualitatively change the results of the paper.

Finally, it is important to point out that I have not included advertising in this model. That is, firms may not randomly contact a fraction $\lambda$ of the consumers at a cost, say $C(\lambda)$, to inform them about their price. Indeed, this has been explored previously by Robert and Stahl (1993), but not in a market where complexity is present. However, the results that follow would not change qualitatively as long as the cost of credibly advertising and educating the entire consumer population is exceedingly expensive. That is, as long as $C(1) = \infty$ (as in Robert and Stahl, 1993), a measure of consumers would still remain uninformed. Therefore, while advertising of this type does occur in reality and might make the industry more competitive, the inability of any one firm to educate all of the consumers in the population implies that the dynamics derived in this paper remain important.

3. Strategic pricing and complexity

In this section, I consider the firms’ problem of creating an industry price structure. I pose the optimization problem faced by the firms and prove existence of a symmetric mixed-strategy Nash equilibrium for the game. I show that the equilibrium involves a positive price mark-up over marginal cost and that a symmetric equilibrium in pure strategies cannot exist. That is, there will always be price dispersion in this industry. Additionally, I derive the
strategic complexity choices that the firms will employ when setting their prices.

Define $J^*$ to be the set of firms who quote the lowest price in equilibrium. Let $n_j$ be the number of firms in $J^*$, so that the $n_j$ firms in $J^*$ split the demand from the experts equally. Each firm $j$ chooses $p_j$ and $k_j$ to maximize its expected profit, that is, they solve

$$\max_{p_j,k_j} \pi_j(p_j,k_j) = p_jQ_j,$$

(2)

where the expected demand $Q_j$ is calculated as

$$Q_j = \mu \frac{1}{n_j} + \frac{1 - \mu}{n}.$$ 

$Q_j$ is composed of two parts. The first expression represents the demand from the fraction $\mu$ of informed consumers. Firm $j$ receives $1/n_j$ of this demand if they are one of the $n_j$ firms that quote the lowest price in the industry. The second expression represents the expected demand from the fraction $1 - \mu$ of uninformed consumers. As such, firm $j$'s choice of $k_j$ affects the proportion of experts $\mu$ and its choice of $p_j$ affects whether they have the lowest price in the industry ($1/n_j$).

The following proposition establishes the existence of a symmetric mixed-strategy Nash equilibrium in this pricing game and characterizes some of its properties.

**Proposition 1 (Existence and characterization).** In the pricing complexity game, there exists a symmetric mixed-strategy Nash equilibrium $\sigma = (F^*(p), k^*(p))$ in which firms choose prices according to the distribution $F^*(p)$ and choose complexity according to the map

$$k^*(p) = \begin{cases} \frac{k}{\hat{p}} & \text{if } p < \hat{p}, \\ k & \text{if } p > \hat{p}, \\ k \in [\hat{k}, \bar{k}] & \text{if } p = \hat{p}, \end{cases}$$

(3)

where

$$\hat{p} = F^{-1}\left(1 - \frac{1}{n}\right)^{1/(n-1)}.$$ 

In equilibrium, the distribution function $F^*(p)$ is continuous and strictly increasing in $p$.

The ex ante probability that each firm chooses high complexity $\bar{k}$ is uniquely determined to be $[1/n]^{1/(n-1)}$. Additionally, the expected fraction of informed consumers $E[1]$ is also uniquely determined in equilibrium.

The proof of Proposition 1 is given in its entirety in the Appendix. The outline of the arguments used there is as follows. Inspecting (2), the payoff function for each firm $j$ is continuous, except when its price is the lowest and is equal to at least one of its competitors. In this case, the firm may discontinuously increase (decrease) its payoff by lowering (raising) its price. It is possible to show, however, that each firm’s payoff function is indeed weakly lower semi-continuous when its price is the lowest and equal to at least one of its competitors. Additionally, since the sum of the payoffs to all of the firms is a continuous function of any one firm’s price, the pricing complexity game satisfies the conditions that are required for the existence of a symmetric mixed-strategy Nash equilibrium as outlined by Dasgupta and Maskin (1986). It is then possible to show that the optimal complexity choice for each firm only depends on its own price and the distribution of prices $F^*(p)$ that competing firms use to mix over prices, as defined in (3). The continuity and monotonicity of $F^*(p)$ follow from similar arguments as in Varian (1980). Finally, I conclude the proof by showing that the ex ante probability of adding high complexity and the expected fraction of informed consumers is uniquely determined in equilibrium and only depend on $n$.

According to Proposition 1, prices are always dispersed in equilibrium since the distribution $F^*(p)$ has no mass points. This type of mixed-strategy is also present in other models of competitive pricing with consumer search (e.g., Varian, 1980; Rosenthal, 1980; Stahl, 1989; Robert and Stahl, 1993). As such, a one-price equilibrium is impossible and perfect competition does not arise in the market. To gain some intuition for this result, consider without loss of generality that all firms except firm $j$ choose a pure pricing strategy such that $p_{-j} = p'$. Then,

$$\pi_j(p_j, k_j) = \frac{p_j}{\hat{p}}$$

for all $k_j \in [\hat{k}, \bar{k}]$. Since for $\epsilon > 0$ arbitrarily small

$$\pi_j(p' - \epsilon, k) > \frac{p'}{\hat{p}},$$

firm $j$ has an incentive to undercut its competitors and lower its complexity to minimize any negative effects it has on the consumer population. The last inequality results from the fact that there is a market share (mass of experts) that firm $j$ no longer has to share.

In several pricing models, this price cutting behavior causes a Bertrand Paradox, that is, prices equal marginal cost in equilibrium. However, a Bertrand Paradox does not arise in this market because marginal cost pricing is a dominated strategy. To see this, suppose that even one of firm $j$’s competitors offers a price $p = 0$. Since

$$\pi_j(p_j, k_j) = \frac{p_j}{\hat{p}}(1 - \mu) > \pi_j(0, k_j)$$

for any $p_j > 0$ and for all $k_j \in [\hat{k}, \bar{k})$, firm $j$ can earn positive expected profits by pricing its product strictly above its marginal cost. Based on this, Proposition 1 implies that there will exist a non-degenerate distribution of prices in equilibrium and an absence of marginal cost pricing.

According to Proposition 1, the firms choose their complexity given their draw from $F^*(p)$. Thus, it is not optimal for any firm $j$ to mix over the entire strategy space $\Sigma_j$, that is, $k^*(\cdot)$ is a deterministic map based on $p$. The intuition behind this result is that each firm only needs to consider their expected relative price ranking, which is determined by the price that they choose and the distribution that competing firms use when they set their prices. For low-priced draws (below a threshold level $\hat{p}$), it is an optimal strategy to choose minimal complexity $k$. For high-priced draws (above $\hat{p}$), it is optimal to choose $\bar{k}$. Intuitively, if a firm has a low price, they want consumers to be informed about it. The low-price firms will choose $\bar{k}$.
to maximize the fraction $\mu$ of expert consumers. In contrast, if a firm has a relatively high price (above $\hat{p}$), they want consumers to be poorly informed. The high-price firms will choose $\bar{K}$ to minimize the expert population. As long as $p \neq \hat{p}$, the firms choose $k_j$ in a binary fashion as in (3). If indeed $p = \hat{p}$, firms are indifferent between any $k \in [\bar{k}, \bar{K}]$. However, since $F'(p)$ is continuous and strictly monotonic, the event that $p = \hat{p}$ is of zero measure.

As such, the ex ante probability that a firm chooses high complexity to disclose their prices is set at

$$1 - F(\hat{p}) = \left[ \frac{1}{n} \right]^{1/(n-1)} ,$$

which only depends on $n$. Since this probability is only a function of the number of firms, the expected proportion of informed consumers (E($\mu$)) is also pinned down by $n$. The cautious reader will probably have realized that Proposition 1 does not imply that there exists a unique equilibrium for the game, that is, that there may be more than one $F'(\hat{p})$ that may exist in equilibrium. However, for any $F'(p)$ that may exist, there will also exist a corresponding cutoff price $\hat{p}$ such that the probability that a firm chooses high complexity remains given by $[1/n]^{1/(n-1)}$. Therefore, even though uniqueness of the equilibrium remains unproven, the probability of making certain complexity choices and the expected fraction of experts is indeed pinned down uniquely.

Given this dependence on $n$, it follows that industry competition not only affects the complexity that is present in the market, but may also affect the knowledge that consumers have when they purchase financial products. These are the topics of the next section.

4. Competition, complexity, and financial literacy

In this section, I study how increasing competition affects the complexity that arises in the market. Given the model posed in Section 2, I show that as more firms compete in the industry, the probability that each firm adds complexity rises. The intuition is as follows. As more firms compete for the market share of the experts, any firm’s chance of winning this business decreases. As a best-response, each firm tends to add more complexity, in an attempt to increase the fraction of consumers who are uninformed and increase their profits in the case that they do not win demand from the experts. Therefore, in aggregate the firms may use complexity to preserve industry rents in the face of higher competition.

The following proposition characterizes the strategic complexity choices that result when industry competition increases.

**Proposition 2 (Competition and complexity).** In the pricing complexity game, the probability that a firm chooses high complexity ($\bar{K}$) is monotonically increasing in the number of firms $n$. In the limit as $n \to \infty$, all firms choose $\bar{K}$.

The results in Proposition 2 can be appreciated analytically as follows. According to Proposition 1, the probability that a firm chooses $k_j = \bar{K}$ is

$$1 - F(\hat{p}) = \left[ \frac{1}{n} \right]^{1/(n-1)} .$$

Since the right-hand side of (4) is increasing in $n$, each firm’s probability of choosing high complexity is monotonically increasing in $n$. Likewise, taking the limit of $1 - F(\hat{p})$ as $n \to \infty$ yields

$$\lim_{n \to \infty} \left[ \frac{1}{n} \right]^{1/(n-1)} \to 1 .$$

Proposition 2 implies that a higher proportion of firms will add complexity to their prices when there is greater competition. Thus, we can draw a novel empirical prediction from the analysis: industry concentration and price complexity should be negatively correlated, ceteris paribus. As noted in the Introduction, this prediction has yet to be tested. There does exist some indirect empirical evidence supporting this claim, however. We know from Hortacsu and Syverson (2004) that entry into the S&P index fund industry between 1995–1999 was associated with a rightward shift in the distribution of prices. Further, in their paper Hortacsu and Syverson estimate the search costs that investors faced to become informed during the period. They found that while these costs decreased for the bottom 85th percentile of the distribution, they increased for investors at the high end. Hortacsu and Syverson posit that this could be explained by the influx of novice investors. Another plausible explanation, however, might be that the market was becoming more challenging to analyze during this period, perhaps due to rising complexity. For example, if new participants in the market did indeed represent the top of the cost distribution and new participants in later years were not inherently less intelligent than new participants in prior years, increasing transparency in the market would imply that search costs should decrease uniformly over time. Since this does not appear to be the case empirically, it could imply that complexity was increasing during that period.

It is important to note that the analytical results in Proposition 2 are based on a map $\mu$ that is given exogenously in the model. As such, Proposition 2 should be considered with some qualification. Specifically, how informed consumers are in the model depends only upon the complexity choices of the firms, not on their equilibrium pricing behavior. Therefore, the tendency for consumers to search for the best alternative does not depend on the amount of price dispersion in the market. An alternative specification might be to explicitly model a continuum of individual decision-makers (consumers) who choose optimally whether to become informed about the industry, given the complexity present and the equilibrium pricing strategies of the firms. Indeed, I have evaluated such a model and unfortunately found it to be limited by intractability; therefore, I have chosen to pursue the more parsimonious model presented in the paper, recognizing its potential limitations.

It should also be pointed out that the relationship between competition and financial literacy remains
equivalently in the model. That is, while financial literacy may suffer as the result of rising competition and complexity, this result is not guaranteed to evolve. Analytically, this depends on the particular map \( \mu \) that we consider. To gain intuition for this, let us consider two examples in which the fraction of experts in the market is a function of the average of the complexity choices for the firms in the market. Consider first that

\[
\mu_1 = 1 - \frac{1}{n} \sum_{j=1}^{n} k_j, \tag{5}
\]

where \( 0 < k < \bar{k} < 1 \). It is straightforward to verify that this map \( \mu \) satisfies the conditions defined in Section 2. Further, consider that \( \bar{k} - k = \alpha \) such that \( \alpha \in (0,1) \). It follows then from (5) that \( \mu_{\text{max}} - \mu_{\text{min}} = \alpha \).

Since each firm’s complexity choice is binary and the probability of adding high complexity is \([1/n]^{1/(n-1)}\), the expected fraction of expert consumers in the market may be calculated as

\[
E[\mu_1] = \mu_{\text{max}} - \frac{1}{n!} \sum_{m=0}^{\min(n,m)} \frac{n!}{m!(n-m)!} \left[ \left( \frac{1}{n} \right)^{1/(n-1)} \right]^m \left( 1 - \frac{1}{n} \right)^{n-m} , \tag{6}
\]

where \( m \) is the number of firms who choose high complexity. By inspection of (6), \( E[\mu_1] \) is computed as \( \mu_{\text{max}} \) minus the expectation of a binomial random variable.

The expected fraction of experts may then be computed as

\[
E[\mu_1] = \mu_{\text{max}} - \frac{\alpha}{n} \left[ \frac{1}{n} \right]^{1/(n-1)} = \mu_{\text{max}} - \frac{\alpha}{n} \left[ \frac{1}{n} \right]^{1/(n-1)} , \tag{7}
\]

which is decreasing in \( n \). Therefore, as competition in the market becomes more severe in this case, the expected number of informed consumers falls. In fact, as \( n \to \infty \)

\[
\lim_{n \to \infty} E[\mu_1] \to \mu_{\text{min}} .
\]

Now, consider in contrast that \( \mu \) takes the form

\[
\mu_2 = 1 - \frac{1}{n^2} \sum_{j=1}^{n} k_j, \tag{7}
\]

where again \( 0 < k < \bar{k} < 1 \) and \( \mu_2 \) satisfies the conditions defined in Section 2. As before, consider that \( \bar{k} - k = \alpha \) such that \( \alpha \in (0,1) \). By construction, \( \mu_2 \) is more sensitive to changes in \( n \) than \( \mu_1 \), which might be the case if \( \mu_2 \) captures the idea that the government adds educational initiatives as the industry grows in size or that consumer interest groups might be expected to arise as the industry evolves.

As before, each firm’s complexity choice is binary and the probability of adding high complexity is \([1/n]^{1/(n-1)}\).

The expected fraction of expert consumers in the market may be calculated as

\[
E[\mu_2] = \mu_{\text{max}} - \frac{1}{n^2} \sum_{m=0}^{\min(n,m)} \frac{n!}{m!(n-m)!} \left[ \left( \frac{1}{n} \right)^{1/(n-1)} \right]^m \left( 1 - \frac{1}{n} \right)^{n-m} . \tag{8}
\]

As such, the expected fraction of experts is then computed as

\[
E[\mu_2] = \mu_{\text{max}} - \frac{\alpha}{n^2} \left[ \frac{1}{n} \right]^{1/(n-1)} = \mu_{\text{max}} - \frac{\alpha}{n^2} \left[ \frac{1}{n} \right]^{1/(n-1)} , \tag{8}
\]

which is increasing in \( n \). Therefore, as competition increases, the expected number of informed consumers increases, despite the increased tendency for firms to add complexity. In this case, as \( n \to \infty \)

\[
\lim_{n \to \infty} E[\mu_2] \to \mu_{\text{max}} .
\]

The two examples highlight that financial literacy is a function of both the complexity choices of the firms, as well as factors outside of the model analyzed in this paper. Therefore, rising competition may have disparate effects on both consumer knowledge and welfare (through prices). Studying other such factors that are present in these markets (e.g., optimal regulation and the role of advisors) is the subject of future research, and would need to be considered in any empirical tests of the theory presented in this paper.

5. Conclusion

Purchasing a retail financial product requires effort. Because prices in the market are complex, consumers must pay a cost (time or money) to compare prices in the market. Some consumers gain sufficient expertise and get the best deal. Those with high search costs forego value, and often make purchases without knowing exactly what they are getting or how much they are paying. In fact, they may also be unaware that they are indeed over-paying (Choi, Laibson, and Madrian, 2006). There is now a large literature that documents these findings (e.g., Capon, Fitzsimons, and Prince, 1996; Alexander, Jones, and Nigro, 1998; Barber, Odean, and Zheng, 2005; Agnew and Szykman, 2005) and this phenomenon may have substantial welfare implications (Campbell, 2006; Calvet, Campbell, and Sodini, 2007).

In this paper, I develop a model of pricing complexity in which firms compete on price for market share and strategically add complexity to preserve market power in the face of competitive pressures. The resulting equilibrium matches empirical observation: price dispersion persists even when goods are homogeneous and prices do not converge to marginal cost despite a large number of firms. The analysis in the paper has important social implications, given the large size of retail financial markets.

The paper also adds to an extensive literature on consumer search (e.g., Diamond, 1971; Salop and Stiglitz, 1977; Weitzman, 1979; Varian, 1980; Carlson and McAfee, 1983; Stahl, 1989). Whereas the search environment is constructed exogenously in most models in this literature, I consider a setting in which the firms can alter the quality of information transmission within the market, and affect the ability of consumers to become knowledgeable about their purchases. The results that I derive capture several stylized features of retail financial markets, as well as markets in which other homogeneous products are sold (e.g., books). Therefore, the paper is of general economic interest as well.

I believe that this paper presents a plausible argument for considering complexity as an important determinant of price formation in retail financial markets. Given the
large number of potential extensions of this analysis, this paper hopefully represents an important step toward greater understanding of the effect of complexity on security design, asset prices, and market structure.

Appendix A

A.1. Proof of Proposition 1

Outline of proof: The payoff function for each firm $j \in N$ is continuous, except when its price is the lowest and equal to at least one of its competitors. In this case, the firm may discontinuously increase (decrease) its payoff by lowering (raising) its price. According to Dasgupta and Maskin (1986), a symmetric mixed-strategy Nash equilibrium is guaranteed in this case as long as two conditions are satisfied:

(i) The sum of the payoffs to all of the firms is upper semi-continuous.
(ii) Each firm's payoff function is weakly lower semi-continuous at the points (actions) in the discontinuity set.

In what follows, I will show that these two conditions hold and by Theorem 6’ in the appendix of Dasgupta and Maskin (1986), a symmetric mixed-strategy Nash equilibrium exists for the game. Following this, I show that the optimal complexity choice for each firm completely depends on its own price and the distribution of prices $F(p)$ that competing firms use to mix over prices, as in (3). I then show that $F(p)$ is continuous and strictly increasing in equilibrium by following similar arguments as in Varian (1980). Finally, I conclude the proof by showing that the expected fraction of informed consumers is uniquely determined in equilibrium.

Proof. Since I prove existence using results derived by Dasgupta and Maskin (1986), I follow their notation. Let $A_j = [0, v] \times [k, K]$ be the action space for firm $j$ and let $a_j \in A_j$ be a price-complexity pair in that space. As such, $A_j$ is non-empty, compact, and convex for all $j$. Define $A = \times_{j \in N} A_j$ and $a = (a_1, \ldots, a_n)$.

Let $U_j : A \to \mathbb{R}$ be defined as the profit function in (2).

Define the set $A^j$ by

$A^j = \{ (a_1, \ldots, a_n) \in A : \exists j \neq j \text{ s.t. } p_j = p_i \}$

and the set $A^{**}$ by

$A^{**} = \{ (a_1, \ldots, a_n) \in A : \exists j \neq j \text{ s.t. } p_j = p_i = p_{\text{min}} > 0 \}$.

As such, the payoff function $U_j$ is bounded and continuous, except over points $a \in A^{**} (j)$.

The sum $\sum_{j \in N} U_j(a)$ is continuous since discontinuous shifts in demand from informed consumers between firms at points in $A^{**} = \times_{j \in N} A^{**} (j)$ occur as transfers between firms who have the same low price in the industry.

To prove the weak lower semi-continuity of $U_j(a_j, a_{-j})$, define $B^2$ to be the surface of the unit circle. Let $e = (e_1, e_2) \in B^2$ and let $\theta > 0$ be a positive number. Define $B_2^e \subseteq B^2$ such that $B_2^e = \{ e: e_1 < 0 \}$. Finally, define $\Omega$ to be the set of absolutely continuous distributions on $B^2$ such that for any point $e$ that is not in $B_2^e$, $\omega(e) = 0$ for all $\omega \in \Omega$. Therefore, for any absolutely continuous measure $\omega \in \Omega$,

$$\int_{B^2} \left[ \liminf_{\theta \to 0} \vert U_j(a_j + \theta e, a_{-j}) \omega(e) \right] \leq U_j(a_j, a_{-j})$$

(A.1)

for all $a_{-j} \in A^{**} (a_j)$. Hence, $U_j(a)$ is weakly lower semi-continuous. Intuitively, since $\omega$ only places positive measure on points with prices less than $p_{\text{min}}$, the limit in the integral is strictly higher than the payoff the firm would receive if it had to share the demand from informed consumers with any of its competitors. Finally, (A.1) holds with strict inequality when $a_j = a_i$ for all $i \in N \setminus j$ (the so-called $\alpha$ property in Dasgupta and Maskin, 1986).

Therefore, according to Theorem 6’ of Dasgupta and Maskin (1986), since $\forall j A_j$ is non-empty, compact, and convex, $U_j : A \to \mathbb{R}$ is bounded and continuous, except over the set $A^{**} = A^{**} (a_j)$, $\sum_{j \in N} U_j(a)$ is continuous, $U_j(a)$ is weakly lower semi-continuous, and $U_j(a)$ satisfies the $\alpha$ property, there exists a symmetric mixed-strategy Nash equilibrium of the game.

Remark 1. The $\alpha$ property is not necessary for a symmetric mixed-strategy Nash equilibrium to exist, but does imply that $\forall a_j \in A^{**} (a_j)$, the probability that firm $j$ plays $\bar{a}_j$ is a measure zero event.

Now we can characterize the equilibrium choice of complexity. Define $I_j(p_j, k_j|\sigma_{-j})$ as the expected profit for firm $j$ when it chooses $p_j$ and $k_j$, given the symmetric mixed strategies of the other firms $\sigma_{-j}$. Further, let us define two other functions:

$I_j(p_j, k_j, \sigma_{-j}) \equiv E_{\sigma_{-j}} [\mu(p_j, k_j)]$

$\Phi_j(p_j, k_j, \sigma_{-j}) \equiv E_{\sigma_{-j}} [\mu(p_j, k_j)^+ - p_{\text{min}}]$.

The function $I_j(p_j, k_j, \sigma_{-j})$ is the conditional expectation of $\mu$ given a choice of $k_j$ and $p_j$ for firm $j$ and the strategies of the other firms. Likewise, $\Phi_j(p_j, k_j|\sigma_{-j})$ is the conditional expectation of $\mu$, given firm $j$'s choice of $p_j$ and $k_j$, the strategies of the other firms, and that $p_j = p_{\text{min}}$. For clarity, I will use $I$ and $\Phi$ in the rest of the proof to represent $I(p_j, k_j, \sigma_{-j})$ and $\Phi(p_j, k_j, \sigma_{-j})$, unless it is necessary to specify its arguments.

Suppose that all firms except for firm $j$ use the strategy $F(p), H(k)$, where $F(\cdot)$ and $H(\cdot)$ are distributions that the firms use when they mix over each dimension of their action space. The expected profit for firm $j$ is

$I_j(p_j, k_j|\sigma_{-j}) = p_j E_{\sigma_{-j}} \left[ \mu 1_{\{p_j = p_{\text{min}}\}} + \frac{1 - \mu}{n} \right]$.
which may be rewritten as

\[ I(p_j; k_j|\sigma_j) = p_j \left[ \Phi(1 - F(p_j))^{n-1} + \frac{1 - \Gamma}{n} \right]. \]

Differentiating \( I(p_j; k_j|\sigma_j) \) with respect to \( k_j \) yields

\[ \frac{\partial I}{\partial k_j} = p_j \left[ 1 - F(p_j) \right]^{n-1} \frac{\partial \Phi}{\partial k_j} - \frac{1}{n} \frac{\partial \Gamma}{\partial k_j}. \]  

(A.2)

Given that \( \frac{\partial^2 \mu}{\partial k_j \partial k_i} = 0 \), this implies that \( \frac{\partial \Phi}{\partial k_j} = \frac{\partial \Gamma}{\partial k_j} \). That is, since the complexity choices of the other firms do not affect the magnitude with which changes in \( k_j \) affect the population of informed consumers, then \( \frac{\partial \Phi}{\partial k_j} \) and \( \frac{\partial \Gamma}{\partial k_j} \) are equal and only depend on firm j’s choice of \( k_j \) (note that this does not mean that \( \Gamma = \Phi \)).

Therefore, (A.2) may be rewritten as

\[ \frac{\partial I}{\partial k_j} = p_j \left[ 1 - F(p_j) \right]^{n-1} \frac{\partial \Phi}{\partial k_j} - \frac{1}{n} \frac{\partial \Gamma}{\partial k_j}. \]

Since \( \frac{\partial \Phi}{\partial k_j} = \frac{\partial \Gamma}{\partial k_j} \), we have

\[ \left[ 1 - F(p_j) \right]^{n-1} \geq \frac{1}{n}, \]

we have \( \frac{\partial I}{\partial k_j} < 0 \) and obtain the corner solution \( k = k_j \).

This occurs when \( p < \hat{p} \), where the threshold level \( \hat{p} \) is

\[ \hat{p} = F^{-1} \left( \frac{1}{n} \right)^{1/(n-1)}. \]

When \( p > \hat{p} \), that is when

\[ \left[ 1 - F(p_j) \right]^{n-1} < \frac{1}{n}, \]

we have \( \frac{\partial I}{\partial k_j} > 0 \) and obtain the other corner solution \( k = k_j \). Therefore, the equilibrium complexity choice for a firm only depends on its choice of \( p \). When \( p \neq \hat{p} \), \( k'(p) \) is uniquely determined by (3), whereas when \( p = \hat{p} \), the firm is indifferent between any \( k \in [k_j, k_f] \).

Now, we can prove properties about \( F^*(p) \).

(i) Continuity: Suppose that there did exist a countable number of mass points in the distribution of \( F^*(p) \). Then, we can find a mass point \( p' \) and an \( \varepsilon > 0 \) such that \( f^*(p') = a > 0 \) and \( f^*(p' - \varepsilon) = 0 \). Now consider a deviation by firm j to choose \( F(p) \) such that \( f(p') = 0 \) and \( f(p' - \varepsilon) = a \). Since \( \text{E}[I_j(p, k)] \) using \( F(p) \) is strictly less than using \( F(k) \), this would be a profitable deviation. Therefore, in equilibrium, no mass points can exist.

(ii) Strict monotonicity (Increasing): Suppose there exists an interval \([p_a, p_b]\) within \( [0, \varepsilon] \) such that \( F(p_a) - F(p_b) = 0 \). Then, for any \( \hat{p} \) such that \( p_a < \hat{p} < p_b \),

\[ \left[ 1 - F(\hat{p}) \right]^{n-1} = \left[ 1 - F(p_a) \right]^{n-1}. \]

Since \( \hat{p} [1 - F(\hat{p})]^{n-1} \geq p_a [1 - F(p_a)]^{n-1} \), then \( F(\hat{p}) \neq F(p_a) \neq 0 \) for any interval \([p_a, p_b]\) within \( [0, \varepsilon] \).

Remark 2. As long as \( \hat{p} \neq \min \text{U}_j(a_j, a_{-j}) \) is continuous at \( \hat{p} \) even though firm j’s complexity choice is discontinuous at that point. Indeed, since each firm is indifferent between choosing any \( k \in [k, k_f] \) when their price is \( \hat{p} \), as long as \( \hat{p} \neq \min \text{U}_j(a_j, a_{-j}) \), then \( \lim_{\hat{p} \to p} U_j(a_j, a_{-j}) = U_j(\hat{p}, k(\hat{p})) \) and \( \lim_{\hat{p} \to \hat{p}^+} U_j(a_j, a_{-j}) = U_j(\hat{p}, k(\hat{p})) \). It follows then that each firm’s profit function is continuous at \( \hat{p} \) as long as this price is not the lowest in the market. In the case that \( \hat{p} = \min \text{U}_j(a_j, a_{-j}) \), this implies that another equally credible approach to proving Proposition 1 would be to first prove that \( k'(p) \) takes the form in (3), and then prove existence of a mixed distribution \( F^*(p) \) over which it is optimal for all firms to choose their price, given that their competitors are doing likewise. Proving existence of a symmetric mixed-strategy Nash equilibrium reduces to a one-dimensional problem and Theorem 6 in Dasgupta and Maskin (1986) guarantees that such an \( F^*(p) \) exists.

Now, we can consider the ex ante probability that a firm will add high complexity \( k \) to their price structure. For any \( F^*(p) \) that may exist, there exists a corresponding threshold \( \hat{p} \) such that with probability

\[ 1 - F^*(\hat{p}) = \left( \frac{1}{n} \right)^{1/(n-1)}, \]

each firm will add high complexity to their prices. Since this probability only depends on \( n \), it is uniquely determined in equilibrium. Based on this, the expected fraction of experts may be written as

\[ \text{E}[\mu] = \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} \mu(m) \left( \frac{1}{n} \right)^{1/(n-1)} \left( 1 - \frac{1}{n} \right)^{m-n}, \]

where \( \mu(m) \) is the fraction of informed consumers when \( m \) firms choose high complexity and \( n - m \) firms choose low complexity. As such, \( \text{E}[\mu] \) is uniquely determined in equilibrium by the probability of each firm choosing \( k \).

\[ \quad \square \]

**Proof of Proposition 2**. Taking the derivative of \( n^{1/(n-1)} \) with respect to \( n \) yields

\[ \left[ \frac{1}{n} \right]^{1/(n-1)} \left[ \frac{n \ln n - n + 1}{(n-1)^2} \right] \]

which is positive for all \( n \geq 2 \). Therefore, \( 1 - F^*(p) \) is strictly increasing in \( n \). Taking the limit

\[ \lim_{n \to \infty} \left[ \frac{1}{n} \right]^{1/(n-1)} \to 1. \]

\[ \quad \square \]

**References**


